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# ANALYTICAL MODEL FOR THE STRENGTH PREDICTION OF HPC CONSISTING OF CEMENTITIOUS COMPOSITES

Mohammad Iqbal KHAN *

* Dr. Assistant Professor; Department of Civil Engineering, King Saud University,
P.O. Box 800, Riyadh 11421, Kingdom of Saudi Arabia
E-mail address: *miqbal@ksu.edu*

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A b s t r a c t
**The paper presents analytical models for compressive strength of high performance concrete. High performance concrete was developed using various cementitious composites to form various binary and ternary blending combinations consisting of normal portland cement, fly ash, slag and silica fume. Compressive strength of concrete containing normal portland cement, fly ash/slag and silica fume at various ages is reported. Based on the experimentally obtained results, analytical prediction models have been developed. These models are based on the quadratic response surface model having predictive capabilities and permitted the optimization of the parameters under study over the experimental domain.**

S t r e s z c z e n i e
**W artykule przedstawiono model analityczny umożliwiający określanie wytrzymałości na ściskanie betonu wysokowartościowego. Badany beton wysokowartościowy wykonano z mieszanek o różnym składzie, z zastosowaniem cementu portlandzkiego normalnego, popiołu lotnego, żużla oraz pyłu krzemionkowego. W artykule zestawiono wartości wytrzymałości na ściskanie badanych mieszanek betonowych o różnym składzie otrzymane w wyniku badań doświadczalnych. Bazując na wynikach badań doświadczalnych opracowano model analityczny. W modelu wykorzystano równanie kwadratowe, w którym możliwy jest dobór parametrów funkcji oraz jej optymalizacja na podstawie badań doświadczalnych.**

K e y w o r d s: **Analytical prediction model; Compressive strength; High performance concrete; Fly ash; Silica fume; Slag.**

## 1. INTRODUCTION

High performance concrete (HPC) should be determined in terms of both strength and durability performance under anticipated environmental conditions. In order to produce HPC a very dense homogeneous concrete microstructure especially in the interface region between hydrated paste and aggregate is required. This densification and homogeneity of the interfacial region is achieved through the incorporation of mineral admixtures which improve concrete microstructure [1, 2]. Therefore, in addition to the three basic ingredients in conventional concrete, i.e., portland cement, fine and coarse aggregates, and water, the production of HPC needs to incorporate supplementary cementitious materials, such as fly ash, slag, silica fume, and chemical admixture. Therefore, HPC can be manufactured involving up to 10 different ingredients whilst having to consider durability properties in addition to strength.

Binary blends of normal portland cement and fly ash have been widely used in concrete construction in various parts of the world for enhancing concrete performance as well as for economical and ecological reasons. In order to derive maximum benefit from cement blending materials, combinations in ternary systems could offer beneficial solutions. For example, the low early-strength development associated with the use of fly ash in concrete could be overcome with the incorporation of a certain amount of silica fume. In return fly ash could enhance the workability otherwise reduced by the silica fume. Hence, the potential synergy between these materials needs to be investigated for the development of high performance concrete.

This investigation was carried out to develop analytical models for compressive strength of high performance concrete. High performance concrete was developed using binary and ternary blending combinations consisting of normal portland cement, fly ash/slag and silica fume. Compressive strength of concrete containing portland cement, fly ash/slag and silica fume at various ages has been reported. Based on the experimentally obtained results, analytical prediction models were developed. These models enabled the establishment of isoresponse contours showing the interactive influence between the various parameters investigated.

## 2. EXPERIMENTAL DETAILS

### 2.1. Materials

Cement Type I complies with the requirements of the ASTM C150, fly ash complying with ASTM C618 Class F, slag complying with ASTM C989 and silica fume complying with ASTM C1240 were used throughout the investigation. Fine and coarse aggregates available in the laboratory were used for this investigation. In order to meet the ASTM C33 grading limits, 60% silica sand and 40% crushed sand was used as fine aggregate. Crushed coarse aggregate comprising of 80% of 20 mm and 20% of 10 mm was used. Both the fine and coarse aggregates were air-dried before use, and allowance was made for absorption when calculating batch weights.

### 2.2. Sample preparation

The slag (Sg) was used at 0, 40, 50, and 60% and the fly ash (FA) was used at 0, 20, 30, and 40%. To these blends 0, 5, 10 and 15% of silica fume (SF) replacement levels were incorporated to make various binary and ternary cementitious combinations. The incorporation of fly ash, slag and/ or SF was used as partial cement replacement. Water to cementitious (w/c) ratios at 0.30, 0.40 and 0.50 were used. Mixing was done in revolving drum mixer in accordance with ASTM C 192. Specimens were cast and compacted in two layers by external vibration in accordance to the ASTM specifications. After casting, the samples were covered with damp hessain and polyethylene sheets for 24 hours. The samples were demoulded the following day and then kept in the standard curing temperature -22±2°C prior to testing. The measurement of the compressive strength was carried out at 7, 28, 90 and 180 days, in accordance with BS 1881: 1983. Triplicate samples were tested for each age and the mean value is reported as the result.

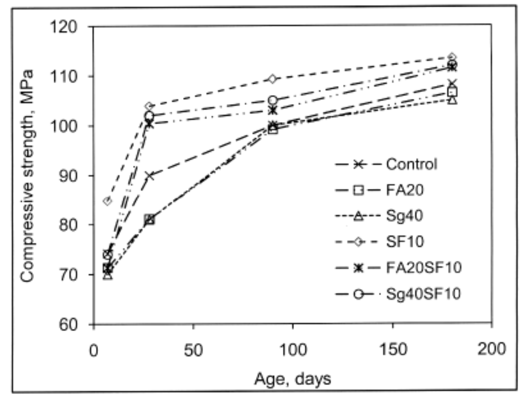


**Figure 1.**
**Compressive strength comparison of 20% FA and 40% Slag incorporating 10% SF (w/c 0.30)**

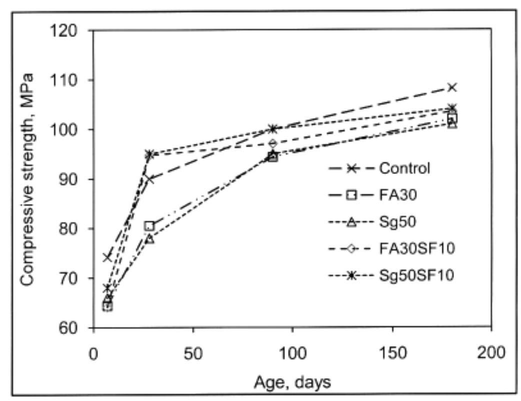


**Figure 2.**
**Compressive strength comparison of 30% FA and 50% Slag incorporating 10% SF (w/c 0.30)**

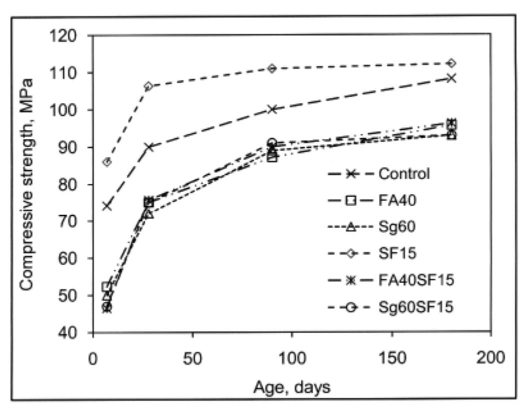


**Figure 3.**
**Compressive strength comparison of 40% FA and 60% Slag incorporating 15% SF (w/c 0.30)**

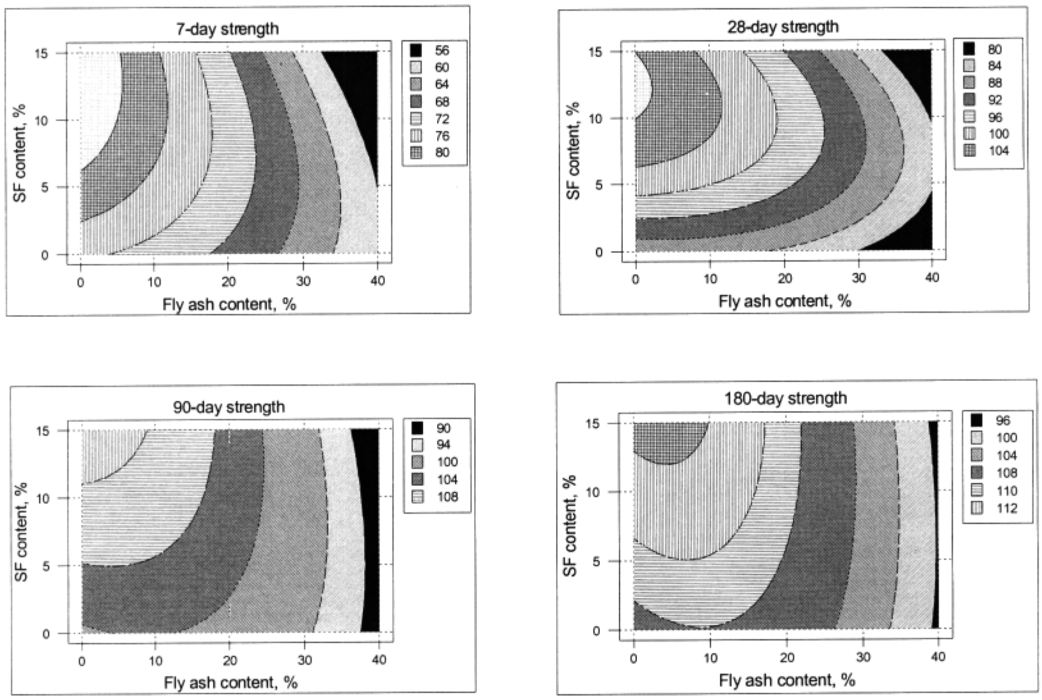


**Figure 4.**
**Predictive model for the compressive strength of concrete containing fly ash and SF (w/c 0.30) at various ages using equation 2**

## 3. RESULTS AND DISCUSSION

Compressive strength measurements were conducted for various binary and ternary concrete systems containing fly ash/slag and silica fume at 7, 28, 90 and 180 days. Comparison of compressive strength of concrete containing fly ash and silica fume system and slag and silica fume system is shown in Figs. 1 to 3. These figures demonstrate similarities of fly ash and slag replacement levels. It can be seen that the compressive strength of 20% fly ash and 40% slag with 10% silica fume or without silica fume content is similar (Fig. 1). Further 20% fly ash and 40% slag showed lower strength at early ages and almost equal strength at later ages as compared to the control mix. However, incorporation of 10% silica fume to these showed higher strength at all ages. Similarly, the compressive strength of 30% fly ash and 50% slag with 10% silica fume or without silica fume content is similar (Fig. 2). Strength of these mixes without silica fume showed lower strength at all ages as compared to the control mix and almost equal when incorporated with 10% silica fume.

Based on the experimentally obtained results, analytical models were developed [3]. These models are based on the quadratic response surface model having predictive capabilities and permitted the optimization of the parameters under study over the experimental domain. A response variable $f(x)$ is

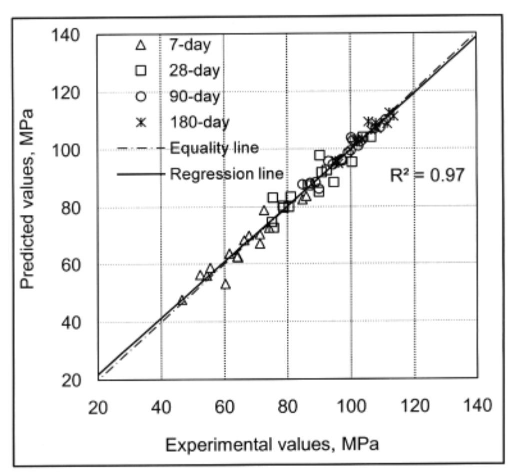


**Figure 5.**
**Relationship between experimental values and predicted values (using equation 2) of the compressive strength of concrete**

measured from combinations of values of two-factor variables $x_1$ (fly ash or slag content) and $x_2$ (silica fume content) using the following model:

$$f(x) = \beta_0 + \beta_0 x_1 + \beta_2 x_2 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{12} x_1 x_2 \quad (1)$$

where $f(x)$ is the observation of the responsive variable; $x_1$ and $x_2$ are the experimental factor variables for $f(x)$; $\beta_0$, $\beta_1$, $\beta_2$, ... , $\beta_{12}$ are the coefficients of the model.

**Table 1.**
**Coefficients of compressive strength model (in equation 2)**

| Age (days) | Coefficients | | | | | | $R^2$ |
|---|---|---|---|---|---|---|---|
| | *a* | *b* | *c* | *d* | *e* | *f* | |
| 7 | 72.66 | - | 1.50 | -0.006 | -0.051 | -0.033 | 0.90 |
| 28 | 85.10 | - | 3.16 | -0.009 | -0.127 | -0.035 | 0.85 |
| 90 | 99.37 | 0.20 | 0.99 | -0.012 | - | -0.021 | 0.93 |
| 180 | 106.88 | 0.23 | 0.55 | -0.013 | - | -0.010 | 0.94 |

**Table 2.**
**Coefficients of compressive strength model (in equation 3)**

| Age (days) | Coefficients | | | | | | $R^2$ |
|---|---|---|---|---|---|---|---|
| | *a* | *b* | *c* | *d* | *e* | *f* | |
| 7 | 71.06 | 0.26 | 2.05 | -0.008 | -0.081 | -0.02 | 0.88 |
| 28 | 85.52 | 0.40 | 2.86 | -0.011 | -0.11 | - 0.02 | 0.85 |
| 90 | 98.92 | 0.51 | 0.72 | -0.012 | - | - 0.013 | 0.92 |
| 180 | 106.57 | 0.40 | 0.66 | -0.011 | - | - 0.006 | 0.90 |

**Table 3.**
**Coefficients of compressive strength model (in equation 4)**

| Age (days) | Coefficients | | | | | | $R^2$ |
|---|---|---|---|---|---|---|---|
| | *a* | *b* | *c* | *d* | *e* | *f* | |
| 7 | 265.28 | -0.93 | -9.33 | - | 0.098 | 0.011 | 0.99 |
| 28 | 298.22 | - | -9.70 | -0.017 | 0.096 | 0.010 | 0.98 |
| 90 | 248.19 | -0.44 | -6.56 | -0.015 | 0.054 | 0.014 | 0.99 |
| 180 | 261.62 | - | -7.10 | -0.017 | 0.062 | 0.007 | 0.99 |

**Table 4.**
**Coefficients of compressive strength model (in equation 5)**

| Age (days) | Coefficients | | | | | | $R^2$ |
|---|---|---|---|---|---|---|---|
| | *a* | *b* | *c* | *d* | *e* | *f* | |
| 7 | 270.92 | -0.14 | -9.60 | -0.01 | 0.10 | 0.005 | 0.99 |
| 28 | 296.45 | - | ˜9.57 | -0.02 | 0.095 | 0.008 | 0.98 |
| 90 | 249.25 | - | ˜6.56 | -0.014 | 0.054 | 0.009 | 0.98 |
| 180 | 269.62 | - | ˜7.50 | -0.013 | 0.066 | 0.006 | 0.98 |

### 3.1. Interaction of fly ash and silica fume

Analytical model for compressive strength of concrete containing combinations of fly ash and silica fume (w/c ratio 0.30) is as follows:

$$f_{cu(n)} = a + bx_1 + cx_2 + dx_1^2 + ex_2^2 + fx_1x_2 \qquad (2)$$

where: $f_{cu(n)}$ – compressive strength at $n$ days age

$x_1$ – fly ash as partial cement replacement (%)

$x_2$ – silica fume as partial cement replacement (%)

The coefficients for 7, 28, 90 and 180 days are shown in Table 1 where $R^2$ is the coefficient of determination and measures the variation in the response which is attributed to the model rather than to the random error. Statistically insignificant terms have been excluded from the equations. These equations were plotted for prediction and optimization purposes as shown in Fig. 4. The predicted values have been plotted against their respective experimentally obtained values as can be seen in Fig. 5. It can be seen from this figure that there is a good correlation between experimental values and those predicted from above the equation.

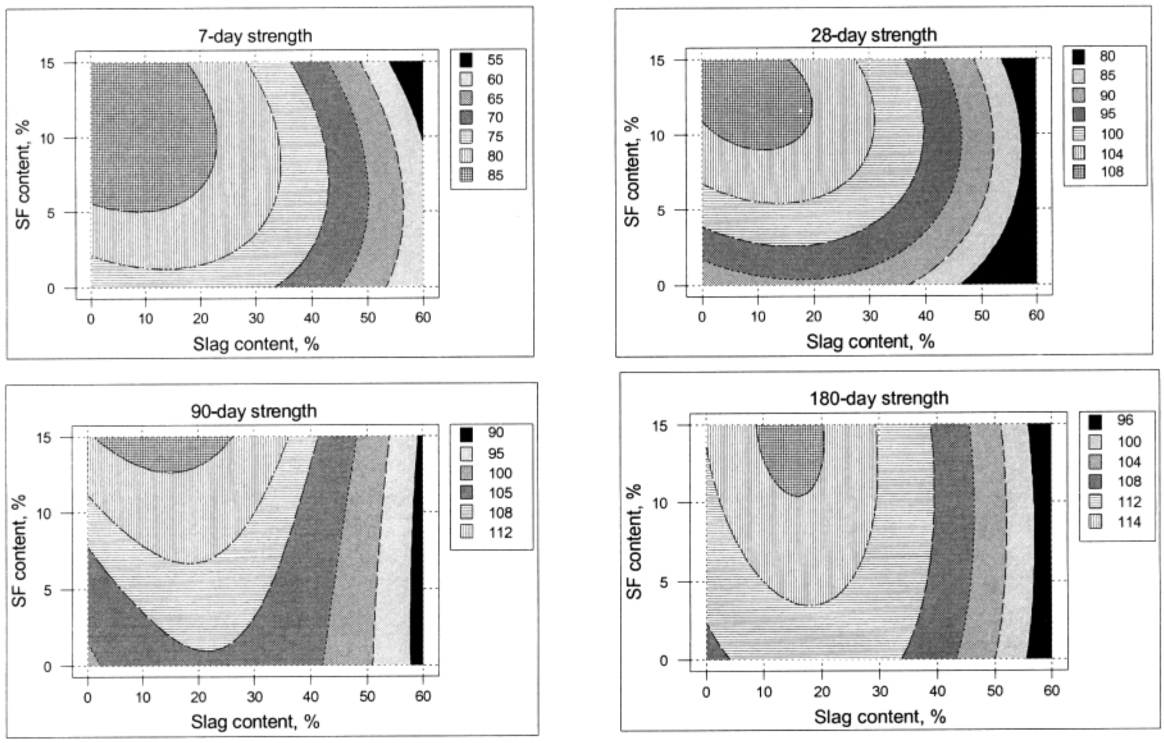


**Figure 6.**
**Predictive model for the compressive strength of concrete containing slag and SF (w/c 0.30) at various ages using equation 3**

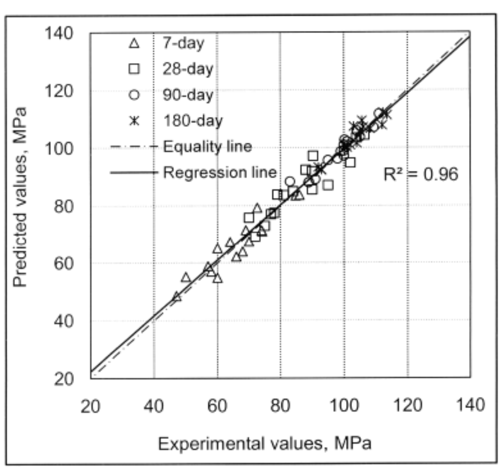


**Figure 7.**
**Relationship between experimental values and predicted values (using equation 3) of the compressive strength of concrete**

It can be seen that compressive strength decreased with an increase in fly ash content for all ages investigated as shown in Fig. 4. At 7 days, silica fume affected the strength of fly ash mixes and this seems to be related to the fly ash content. An increase of strength is registered for fly ash levels lower than 10% when silica fume is incorporated, however, the results suggest that at higher fly ash levels (>30%) the incorporation of silica fume results in a reduction in strength. At the age of 28 days, up to 10% silica fume increased the strength for all levels of fly ash replacements, whilst silica fume above 10% did not result in any advantage in improving the strength and a slight drop in strength was evident when silica fume was incorporated at 15%. At 90 and 180 days, only a modest improvement in strength has resulted from silica fume incorporation and this was evident for low levels of fly ash (<10%) only. The curves for mixes with fly ash content higher than 20% are almost vertical lines, indicating that the presence of silica fume no longer has an influence on the strength.

### 3.2. Interaction of slag and silica fume

Analytical model for compressive strength of concrete containing combinations of slag and silica fume (w/c ratio 0.30) is as follows:

$$f_{cu(n)} = a + bx_1 + cx_2 + dx_1^2 + ex_2^2 + fx_1x_2 \tag{3}$$

where: $f_{cu(n)}$ – compressive strength at $n$ days age

$x_1$ – slag as partial cement replacement (%)

$x_2$ – silica fume as partial cement replacement (%)

The coefficients for 7, 28, 90 and 180 days are shown

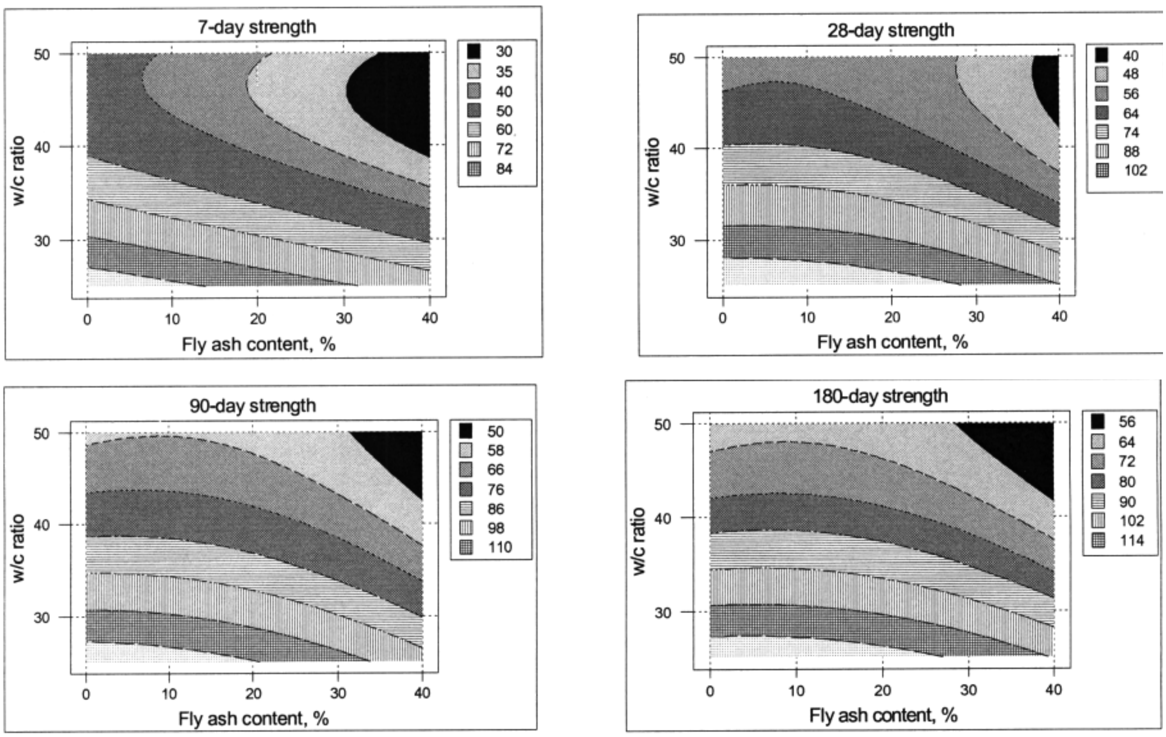


**Figure 8.**
**Predictive model for the compressive strength of concrete containing fly ash and 10% SF influenced by w/c ratio at various ages using equation 4.**

in Table 2. These equations were plotted for prediction and optimization purposes as shown in Fig. 6. There is a good correlation between experimental values and those predicted from the above equation (Fig. 7).

It can be seen that the pattern of the strength development in Fig. 4 and Fig. 6 are almost similar. However, in Fig. 6, slag is used instead of fly ash. The percentage replacements of slag are higher than that of fly ash while demonstrating the similar type of results. As slag is incorporated the strength of concrete remains almost similar at the dosage of 40% slag. After that the strength tends to decrease at the dosages of 50% and 60% of slag. This is similar for the concrete containing fly ash dosages of 30% and 40%.

The incorporation of silica fume to the slag mixes increased the strength and this was related to both slag and silica fume replacement levels. At the age of 28 days, up to 10% of silica fume increased the strength for all levels of slag replacements similar to that in case of fly ash, whilst silica fume above 10% did not result in any advantage in improving the strength. At 90 and 180 days, similar modest improvement in strength has been recorded as in case of fly ash. The curves for mixes with higher than 40% slag are almost vertical line which is similar to that of mixes containing fly ash above 20%, indicating that the presence of silica fume no longer has an influence on the strength.

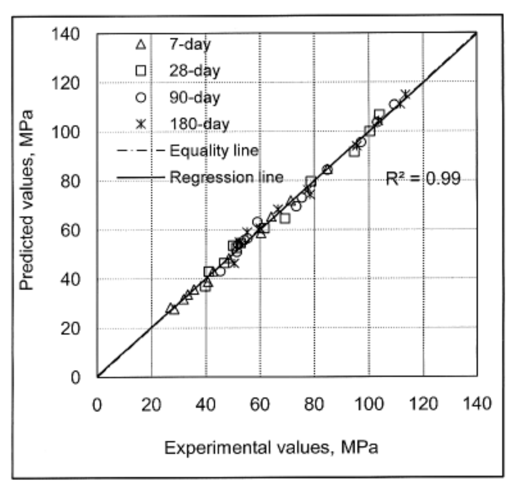


**Figure 9.**
**Relationship between experimental values and predicted values (using equation 4) of the compressive strength of concrete**

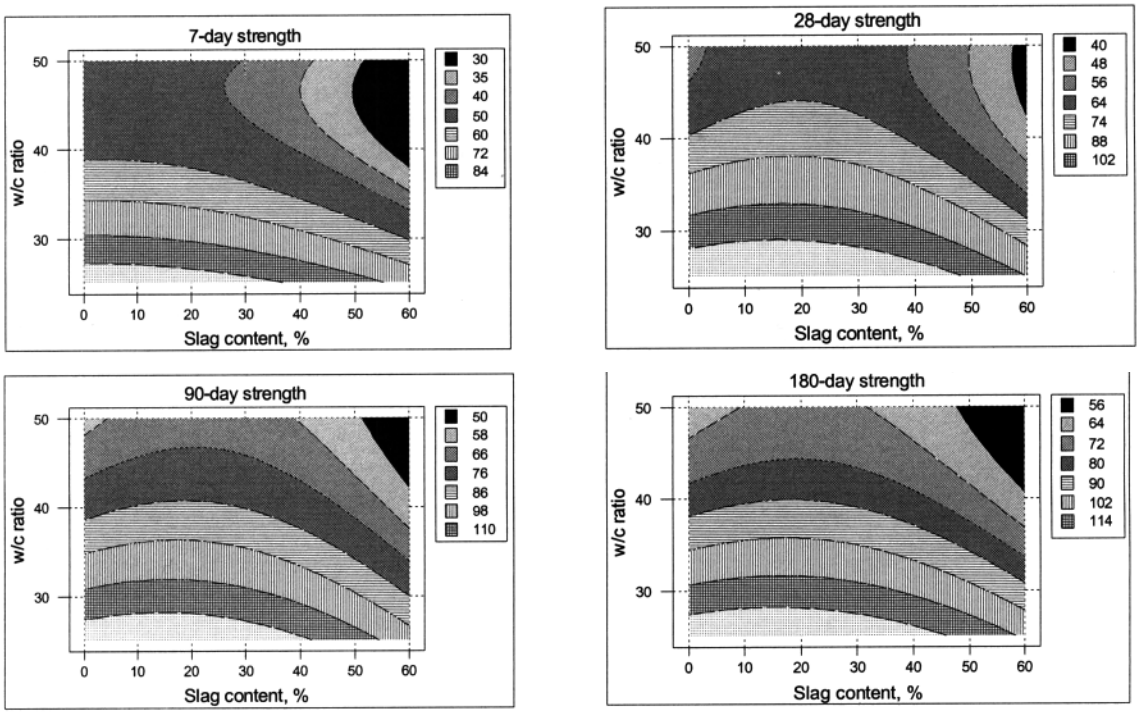


**Figure 10.**
**Predictive model for the compressive strength of concrete containing slag and 10% SF influenced by w/c ratio at various ages using equation 5**

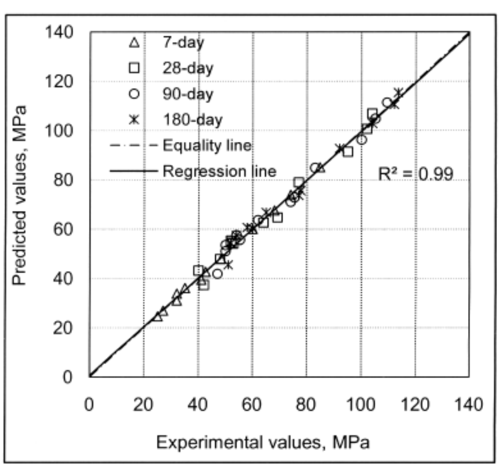


**Figure 11.**
**Relationship between experimental values and predicted values (using equation 5) of the compressive strength of concrete**

### 3.3. Influence of w/c ratio

Fly ash and silica fume composition: The influence of w/c ratio on the compressive strength of concrete containing fly ash and 10% of silica fume is represented by equation 4. The coefficients for the equation are shown in Table 3.

$$f_{cu(n)} = a + bx_1 + cx_3 + dx_1^2 + ex_3^2 + fx_1x_2 \quad (4)$$

where: $f_{cu(n)}$ – compressive strength at $n$ days age

$x_1$ – fly ash as partial cement replacement (%)

$x_2$ – w/b ratio ×10$^2$

The isoresponse curves for the compressive strength of concrete containing fly ash and 10% silica fume are shown in Fig. 8. There is a good correlation between experimental values and those predicted from the above equation (Fig. 9). The strength of concrete decreased with increasing w/b ratio for all ages, as it was expected. It can be seen from Fig. 8 that at 7 days the compressive strength is influenced by both w/b and fly ash content. As curing age increases, however, the reduction in strength with increasing fly ash content becomes less apparent, especially for fly ash contents <30%. At 180 days and w/b ratio <0.45, the reduction of strength with increasing fly ash content is small compared with that for w/b ratio >0.45. As silica fume is incorporated at 10%, the overall level of strength for a given age / fly ash content is increased. The results also show that for >20% fly ash content, the influence of the w/b ratio becomes more apparent than when silica fume is not incorporated.

Slag and silica fume composition: The influence of w/c ratio on the compressive strength of concrete containing slag content and 10% of silica fume is represented by equation 5. The coefficients for the equation are shown in Table 4.

$$f_{cu(n)} = a + bx_1 + cx_3 + dx_1^2 + ex_3^2 + fx_1x_2 \quad (5)$$

where: $f_{cu(n)}$ – compressive strength at $n$ days age

$x_1$ – slag as partial cement replacement (%)

$x_2$ – w/b ratio $\times 10^2$

The isoresponse curves for compressive strength of concrete containing slag and 10% silica fume are shown in Fig. 10. There is a good correlation between experimental values and those predicted from above equation as shown in Fig. 11.

It is evident from Figs. 8 and 10 that the pattern of strength development of fly ash and slag are similar irrespective of presence of silica fume content. The trend obtained in w/c ratios of 0.40 and 0.50 are similar to that of concrete produced with 0.30 w/c ratio. The incorporation of silica fume to the slag mixes increased the strength and this was related to both slag and silica fume replacement levels.

The results indicate that early-age loss of strength as a result of incorporating fly ash and slag was compensated by the inclusion of silica fume to an extent depending on the quantity of fly ash or slag and silica fume. Similar type of influence was found in paste systems using fly ash and silica fume published elsewhere [4, 5]. The gain in strength as a result of silica fume inclusion is attributed to its refined pore structure [6], high pozzolanicity, and its extreme fineness [7].

It is worth noting here that in this investigation the incorporation of 8 to 12% silica fume as cement replacement yielded the optimum performance, resulting in the highest compressive strength values for all levels of fly ash and slag. The results indicate that there is an interaction between fly ash or slag and silica fume, with their level of replacement and the age of curing. The higher values of strength are, as a result of micro-filling and pore refinement, in these mixes and the incorporation of silica fume with or without the presence of fly ash or slag demonstrated significant improvement of pore refinement at all ages as compared to the control mix [8].

## 4. CONCLUSIONS

1. The slow early-strength development associated with the inclusion of fly ash and slag can be reduced with the inclusion of silica fume, but this is generally restricted to low levels of fly ash and slag.
2. Concrete mixes containing above 30% fly ash and above 50% slag, with or without silica fume, were not able to achieve the strength of the control mix. However, these systems are viable given the level of performance achieved where environmental benefits are concerned.
3. The incorporation of 5% silica fume has no significant effect on the compressive strength but the rate of gain in strength is significantly increased with 10% silica fume replacement level. The results suggest that the optimum silica fume replacement for improvement in strength lie between 7 to 12%.
4. The models and interactive figures can provide reasonable predictions of compressive strength, from the knowledge of fly ash or slag and silica fume contents.

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