MODEL OF VIBRATIONS DAMPING AND FRICTION IN JOINT SURFACE OF COMPOSITE CONCRETE STRUCTURES LOADED WITH BENDING MOMENT

Krzysztof GROMYSZ*

*Faculty of Civil Engineering, The Silesian University of Technology, Akademicka 5, 44-100 Gliwice, Poland
E-mail address: *krzysztof.gromysz@polsl.pl*

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Abstract

The subject of analysis is reinforced concrete composite structure. In the paper a model of joint surface between two concretes is assumed. According to this model, when the boundary value of horizontal stresses is achieved, displacement in the joint surface appears. Simultaneously frictional stresses in the joint surface appear. Further increase of vertical force does not result in increase of frictional stresses. Frictional stresses are still equal to boundary value of horizontal stresses. Based on this model equation of free vibration of concrete structure is built. In the discussed model frictional forces in joint surface as well as material damping is taken into consideration. Numerical testing of the model shows that delamination in joint surface affects frequency of vibration as well as damping of vibration of composite concrete-concrete construction.

Streszczenie

Przyjęto model pracy powierzchni zespolenia w zespolonych konstrukcjach betonowo-betonowych obciążonych momentem zginającym i siłą poprzeczną. W modelu tym założono, że po wystąpieniu naprężeń rozwarstwiających o wartości granicznej w powierzchni zespolenia następują przemieszczenia. Przemieszczeniom tym towarzyszy występowanie naprężeń tarcia, których wartość jest równa naprężeniom granicznym. Naprężenia tarcia powodują tłumienie drgań swobodnych konstrukcji. Ponadto drgania swobodne są tłumione przez niesprężyste mikroodkształcenia betonu uwzględnione w badanym modelu jako tłumienie wiskotyczne. W oparciu o powyższe założenia zamodelowano drgania swobodnie podpartej płyty żelbetowej jako układu o jednym stopniu swobody. Zdefiniowane zadanie zostało rozwiązane w sposób analityczny w przypadku szczególnego układu danych. Rozwiązanie dla dowolnego układu parametrów przeprowadzono w sposób numeryczny. Uzyskane wyniki odniesiono do drgań płyty monolitycznej. Wykazano, że występowanie rozwarstwienia w konstrukcji wpływa na zmianę okresu drgań własnych oraz na tłumienie drgań. Powyższe spostrzeżenie może być wykorzystywane do określania stanu powierzchni zespolenia w zespolonych konstrukcjach betonowo-betonowych na podstawie pomiaru drgań swobodnych.

Keywords: **concrete structures; composite structures; vibration damping; free vibrations.**

1. INTRODUCTION

Reinforced concrete beams and slabs consist of prefabricated element and concrete topping cast in situ [1]. Prefabricated element incorporating main reinforcement functions as a stay-in-place formwork. Vertical reinforcement joins precast element and concrete topping. The reinforcement in slabs has a form of truss bracing [1] and in case of composite beams a form of stirrups.

Stiffness of composite reinforced concrete structures loaded with bending moment is determined as for monolithic elements, unless a delamination occurred in joint surface [2]. Stiffness of structure changes as an effect of delamination between prefabricated element (bottom layer of concrete) and concrete topping (top layer of concrete).

The paper assumes a model of work of joint surface of two layers of concrete. Equations of free vibrations of composite reinforced concrete structure loaded with bending moment were calculated on the basis of this model and an assumption of constant, elastic features of two layers of concrete.

2. DEFINITION OF THE MODEL

The considered structure consists of two combined layers of concrete: the lower layer of h_d height and the upper layer of h_g height (Fig. 1a). The structure is vertically loaded. Reactions to this load are: transverse force V and bending moment M. The transverse force causes delaminating stress τ in joint surface, which according to code [2] and [3], in case of combined concrete structures, is related to transverse force in equation

$$\tau(V) = \frac{V}{bz}, \tag{1}$$

where b is the width of cross-section and z is an arm of internal force.

It is important to mention, that in case of elastic structures delaminating stress τ is expressed by formula $\tau = VS/Ib$, where S is static moment of part of the section and I is moment of inertia of the whole section. In this paper, referring to concrete structures, formula (1), derived in [3], is obligatory.

The model assumes a limiting value of horizontal stress τ_{max} in joint surface, which is caused by transverse force V_{roz}. Delamination occurs when the transverse force reaches value V_{roz}. The effect of the delamination is a displacement of prefabricated element in relation to concrete topping. The model assumes that friction stress equal to limiting horizontal stress τ_{max} occurs when the force in joint surface exceeds V_{roz} (Fig. 1b). This stress can be maintained thanks to normal stress in joint surface, compensated by vertical reinforcement. Reduction of transverse force to below V_{roz} value entails horizontal stress coming back to the value resulting from equation 1 (Fig.1b):

$$\tau = \begin{cases} \tau(V),\ when\ |V| \le V_{roz} & \text{(horizontal stress)} \\ \tau_{\max},\ when\ V > V_{roz} & \\ -\tau_{\max},\ when\ V < -V_{roz} & \text{(frictional stress).} \end{cases} \tag{2}$$

Assumed simplified model omits issue of hysteresis of the system and residual stress in joint surface resulting from the strain relief. Moreover, it assumes that the concrete of both bottom and top layers is linear-elastic material characterised by elasticity modulus E_c.

2.1. Model of damping

It is assumed that dissipation of energy occurs as an effect of work of frictional stress τ_{max} in displacement in joint surface. The work is performed on condition that

$$|V| \ge V_{roz} \tag{3}$$

is true.

Stresses τ_{max} are acting on arm e_d in relation to bottom layer of concrete and on arm e_g in relation to top layer of concrete (Fig.1a). These stresses may be substituted with uniformly distributed bending moment, whose value is represented by (Fig. 1c)

$$m = (e_d + e_g) b \tau_{\max}. \tag{4}$$

Further analysis is restricted to composite simply-supported element of l span (Fig. 1c). Deflection of delaminated element in the centre of the span, caused by the bending moment (4) is equal

$$y = \frac{ml^3}{24E_c(I_d + I_g)}, \tag{5}$$

where I_d, I_g are respectively inertia moments of bottom and top layers of concrete. Next part of the analysis will concern a one freedom degree system. Displacements in the system will be represented by vertical displacements of the centre of span of simply supported element. The value of substitute force R, replacing the action of stress τ

$$R = hb\tau_{\max} \tag{6}$$

was calculated from equality of displacements in the centre of span two elements: presented in Fig 1c and Fig. 1d. In the above formula h is a sum of heights h_d and h_g. Force R will occur on condition that deflection y exceeds value equal to transverse force causing displacement in joint surface

$$y_{roz} = \frac{\tau_{\max} l^3 bz}{24E_c}. \tag{7}$$

Force R causes dissipation of energy and because of this it is called damping force.

It is also assumed that dissipation of energy occurs in both top and bottom layers of concrete as an effect of non-elastic micro-strain of the material. Damping related to non-elastic micro-strain occurs irrespectively of the value of transverse force V. The model assumes that the damping meets conditions of viscous damping, i.e. is proportional to velocity $\dot{y}$. Coefficient of viscous damping c is coefficient of proportionality.

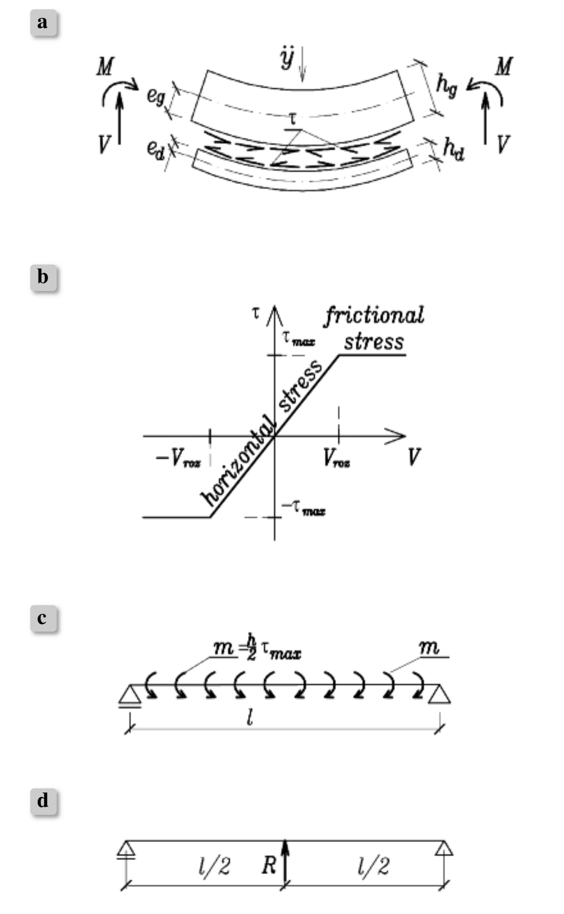

Figure 1.
Assumptions of the model of vibrations damping a) frictional stress in joint surface b) horizontal stress (τ) and frictional stress (τ_{max}) as a function of transverse force c) frictional stress interaction as a uniformly distributed bending moment *m*, d) damping force *R* equivalent to frictional stresses interaction

2.2. Structure model as a one degree freedom system

Simply supported composite element is modelled as a one-degree freedom system. Equivalent mass [4]

$$m = \frac{17}{35} bhl\rho \tag{8}$$

was calculated on the basis of equality of kinetic energy of simply supported beam, whose span is l, cross section b/h, mass density ρ (Fig.2a) and kinetic energy of system, which is concentrated mass attached to the centre of span of weightless beam (Fig.2b).

Analysed system is characterised by two types of stiffness: k_{mon} of non-delaminated element and k_{roz} of delaminated element (Fig.2c)

$$k(y) = \begin{cases} k_{mon} = \dfrac{48E_c I}{l^3}, & for\ |y| \le y_{roz} \\ k_{roz} = \dfrac{48E_c(I_d + I_g)}{l^3}, & for\ |y| > y_{roz} \end{cases}. \tag{9}$$

In the above formula I is moment of inertia of cross-section b/h. As it was mentioned before, damping force R occurs on condition that deflection exceeds y_{roz} value. Moreover, sense of the vector of damping force R is always opposite to sense of the vector of system velocity $\dot{y}$, (Fig. 2d). This means that damping force R is a function of position and stiffness of the system, as in the formula

$$R(y,\dot{y}) = R\frac{1}{2}\left[sign(y - y_{roz}) + sign(y + y_{roz})\right]sign(\dot{y}). \tag{10}$$

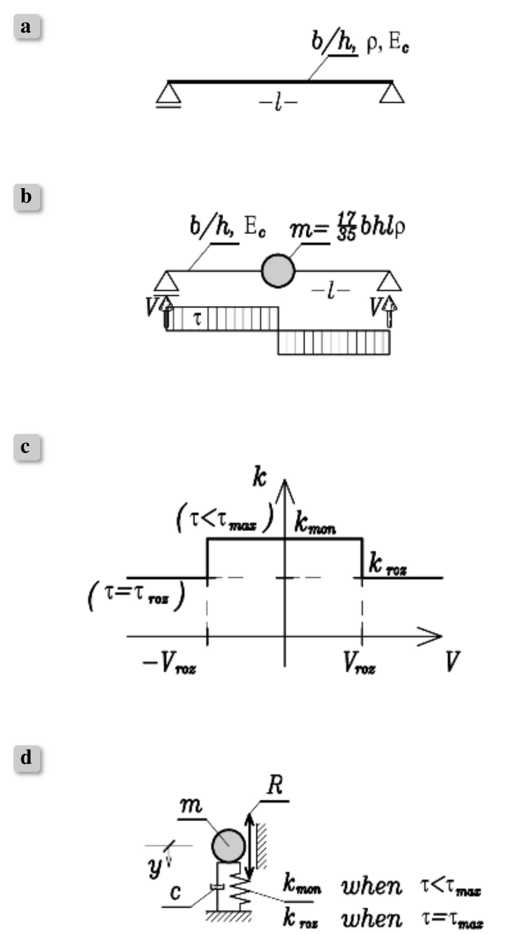

Figure 2.
a) analysed system, b) substitute system, c) discretely variable stiffness of the system in relation to value of transverse force, d) damping elements in the system

Remaining forces interacting with the system are elasticity force equal

$$k(y)\cdot y \tag{11}$$

and force of viscous resistance

$$c\dot{y}\,. \tag{12}$$

According to d'Alembert principle the sum of the above mentioned forces (10÷12) is equal to force of inertia. Thus the equation of system has the following form

$$m\ddot{y} = -c\cdot\dot{y} - k(y)\cdot y - R(y,\dot{y})\,. \tag{13}$$

Further transformation of equation (13) is conducted according to the method universally employed for solving linear differential equations of second degree [5]. Accordingly, if the above equation is divided by m and if we assume

$$2n = \frac{c}{m}\,,\ \omega_0^2 = \frac{k(y)}{m} \tag{14}$$

the following formula is obtained:

$$\ddot{y} + 2n(y)\cdot\dot{y} + \omega_0^2(y)\cdot y + \omega_0^2(y)\frac{R(y,\dot{y})}{k} = 0\,, \tag{15}$$

where

$$\omega_0(y) = \omega_0\frac{1}{2}\left|sign(y+y_{roz}) - sign(y-y_{roz})\right| +$$

$$+\,\omega_{0roz}\frac{1}{2}\left|sign(y-y_{roz}) + sign(y+y_{roz})\right|\ ,$$

$$\omega_{0mon} = \sqrt{\frac{k_{mon}}{m}}\,,\ \omega_{0roz} = \sqrt{\frac{k_{roz}}{m}}\ , \tag{16}$$

$$2n(y) = 2n_{mon}\frac{1}{2}\left|sign(y+y_{roz}) - sign(y-y_{roz})\right| +$$

$$+\,2n_{roz}\frac{1}{2}\left|sign(y-y_{roz}) + sign(y+y_{roz})\right|\ .$$

3. ANALYTIC SOLUTION OF SIMPLIFIED MODEL

Solution of equation (15) will be conducted for numerical values in the next part of the paper. Below the equation is solved analytically, based on assumption that

$$y_{roz} = \infty\,. \tag{17}$$

This means that delaminating does not occur under any loading. In such situation equation (10), with temporary assumption of $sign(\dot{y}) = -1$, is non-homogenous linear differential equation with constant coefficients

$$\ddot{y} + 2n\cdot\dot{y} + \omega_0^2\cdot y = \omega_0^2\frac{R}{k}\,. \tag{18}$$

The solution of homogenous equation (i.e. equation obtained by equating left member of equation (18) with zero) is [6]

$$y_0 = e^{-nt}\left(C_1\cos\sqrt{\left(\omega_0^2 - n^2\right)}\,t + C_2\sqrt{\left(\omega_0^2 - n^2\right)}\,t\right). \tag{19}$$

Solution in stationary state of non-homogeneous equation (18) (in this case t=∞) will correspond to the right member of equation (18). Thus the particular differential of non-uniform equation (18) was determined by method of prediction, assuming that it is a constant function in the form

$$y_1 = C_3\frac{R}{k}\,.$$

After determining consecutive derivatives and substituting them to equation (18) $C_3 = 1$ was obtained.

The final solution of equation (18) is function

$$y = e^{-nt}\left(C_1\cos\sqrt{\left(\omega_0^2 - n^2\right)}\,t + C_2\sin\sqrt{\left(\omega_0^2 - n^2\right)}\,t\right) + \frac{R}{k}\,. \tag{20}$$

Assuming initial conditions

$$y(t=0) = y_0\,,\ \dot{y}(t=0) = 0 \tag{21}$$

constants of solution (20) are determined:

$$C_1 = y_0 - \frac{R}{k}\,,\ C_2 = \frac{ny_0}{\sqrt{\omega_0^2 - n^2}}\ .$$

Substitution of new constants

$$\begin{cases} C_1 = A_0\sin\varphi \\ C_2 = A_0\cos\varphi \end{cases}$$

The solution (20) is transformed to the form

$$y = A_0 e^{-nt}\sin\left(\sqrt{\omega_0^2 - n^2}\ t + \varphi\right) + \frac{R}{k}\,, \tag{22}$$

where

$$A_0 = \sqrt{\left(y_0 - \frac{R}{k}\right)^2 + \frac{(ny_0)^2}{\omega_0^2 - n^2}} \tag{23}$$

$$tg\varphi = \frac{\left(y_0 - \frac{R}{k}\right)\sqrt{\omega_0^2 - n^2}}{ny_0}\,.$$

The above equation results in value $tg\varphi \to \infty$, which means that, when viscous friction $n=0$, the motion of the system is defined by equation

$$y = \left(y_0 - \frac{R}{k} \right) \cos \omega_0 t + \frac{R}{k}. \qquad (24)$$

When $\omega_0 t = \pi$, then velocity $\dot{y} = 0$, which means that the system reaches extreme position A_1 (Fig. 3a). Deflection of the system is then

$$A_1 = -y_0 + 2\frac{R}{k}. \qquad (25)$$

while in extreme position A_2 deflection of the system is

$$A_2 = y_0 - 4\frac{R}{k}. \qquad (26)$$

Amplitude of damped vibrations is limited by a line represented by equation

$$y = y_0 - \frac{2R\omega_0}{k\pi} t, \qquad (27)$$

while angular frequency of free vibrations is ω_0. After the vibrations faded the system, in general, does not accept neutral position (Fig. 3a).

When the damping force $R=0$, a generally known formula for movement of the system with one freedom degree with viscous damping at initial conditions (21) is obtained [7], [8]:

$$y = \sqrt{(y_0)^2 + \frac{(ny_0)^2}{\omega_0^2 - n^2}} e^{-nt} \sin\left(\sqrt{\omega_0^2 - n^2}\, t + \varphi \right), \qquad (28)$$

where

$$tg\varphi = \frac{\sqrt{\omega_0^2 - n^2}}{n}. \qquad (29)$$

Graphic interpretation of the solution is presented in Fig. 3b. In this case the system in stationary state accepts neutral position.

Fig. 3c presents movement of the system in which the damping is composition of damping presented in Fig. 3a (damping in form of constant damping force) and Fig. 3b (viscous damping).

In order to determine value of parameter n, which describes viscous damping, the values of logarithmic decrement of damping Δ were assumed according to [6]. It is generally known that

$$\Delta = \ln \frac{A_n}{A_{n+1}}. \qquad (30)$$

According to (30) and Fig. 3b we may assume that

$$\Delta = \ln \frac{A_{0c} e^{-nt}}{A_{0c} e^{-n\left(t + \frac{2\pi}{\sqrt{\omega_0^2 - n^2}} \right)}} = e^{\frac{2\pi n}{\sqrt{\omega_0^2 - n^2}}}. \qquad (31)$$

As an effect of transformations formula

$$n = \frac{\omega_0 \cdot |\ln \Delta|}{\sqrt{4\pi^2 + \ln^2 \Delta}}. \qquad (32)$$

was obtain.

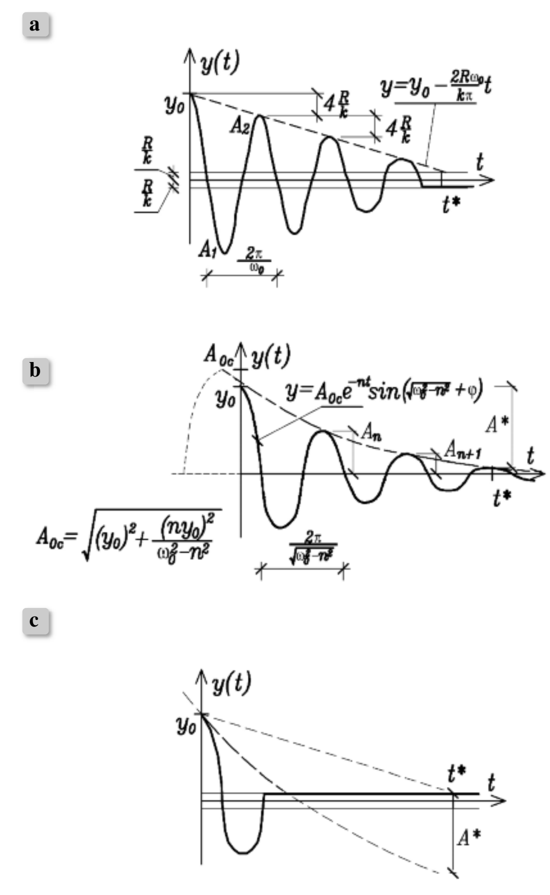

Figure 3.
Solution of system (15) a) solution for $n=0$ and $R\neq0$, b) solution for $n\neq0$ and $R=0$, c) solution for $n\neq0$ and $R\neq0$

The analysis of vibrations of simply supported slab, made of concrete, whose elasticity modulus E_c=27GPa, specific gravity ρ=25kN/m^3, cross-section b/h 0.18/0.59m, span l=2m and initial conditions

$$y(0) = 0.001m,\ \dot{y}(0) = 0 \tag{33}$$

has been presented below.

Assumption (17) is in force, which means, that analysed structure is a monolithic structure. Determined stiffness of the system for the above data is k=56.45MN/m, angular frequency of free vibrations according to (14) is ω_0=420.27rad/s and substitute mass according to (8) is m=262.99kg. Coefficient n, determined from formula (32) based on assumption that Δ=0.15 [6] is 60.74s^{-1}. Fig. 4a presents solution of (18) at initial conditions (33) obtained for constant damping force R=1.06kN and coefficient of viscous damping c=0. Natural period is 14.95ms and temporal course of vibration is limited by a line represented by equation y=0.001-0.006117t.

Fig. 4b presents solution of (18) at initial conditions (33) at n=60.74s^{-1} and R=0. Natural period of the system is 15.11ms and is greater than vibrations of the system with damping by constant force R. The vibrations are represented in equation

$$y = 0.0010106e^{-60.738t}\sin(415.86t + 0.4538\pi).$$

The next point of the paper will make use of the fact that the above presented solution corresponds to simply supported monolithic reinforced concrete slab

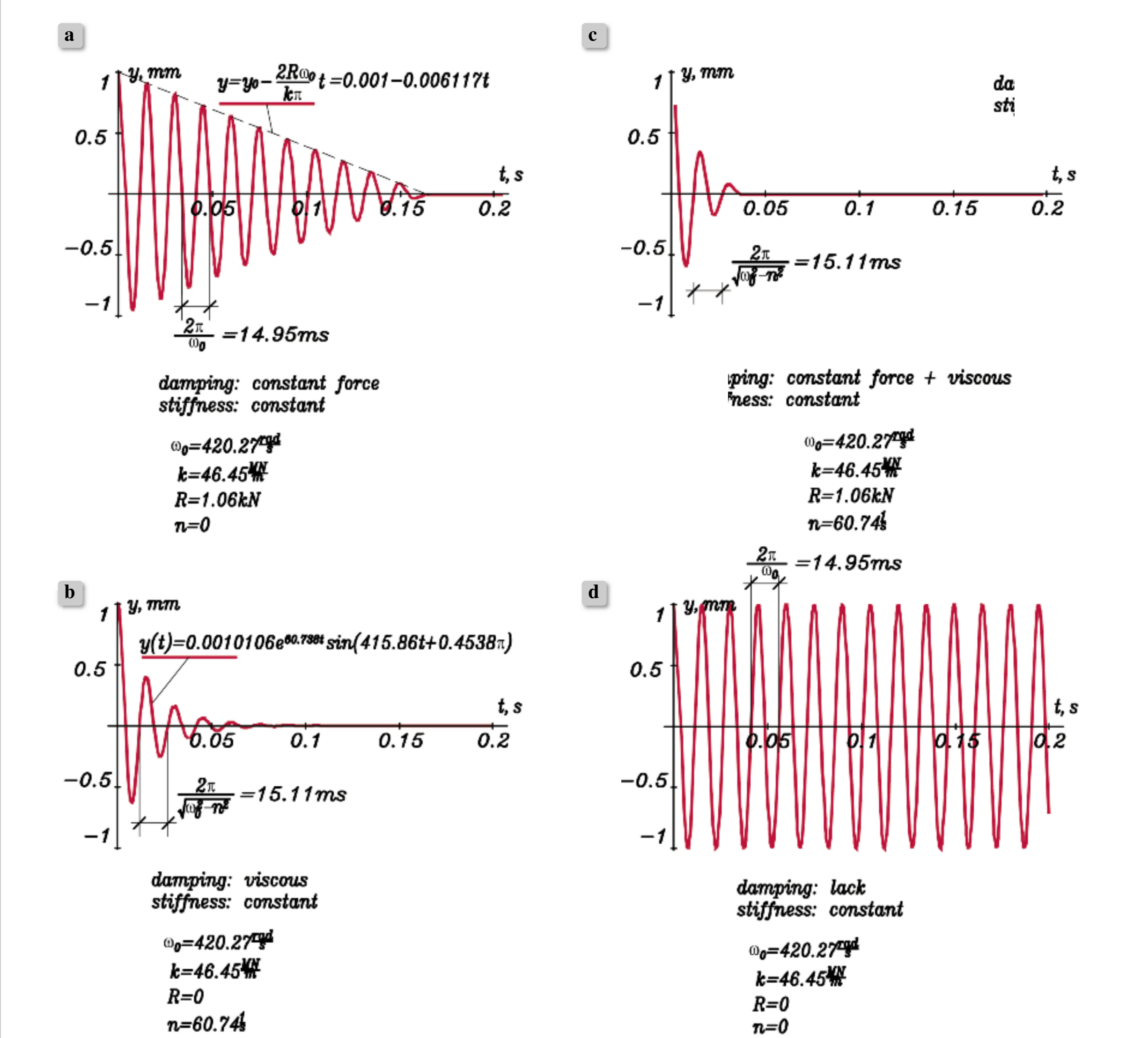

Figure 4.
Results of analythical solutions of equation (18) at assumption $y_{roz}=\infty$ a) solution for n=0 and $R\neq0$, b) solution for $n\neq0$ and R=0, c) solutionfor $n\neq0$ and $R\neq0$, d) solution for n=0 and R=0

$b/h/l$ 0.59/0.18/2m unbalanced by a 1mm displacement in the middle of its span.

Fig. 4c presents solution of equation (18) at identical initial conditions, constant damping force R=1.06kN and parameter of viscous damping n=60.74s^{-1}. Natural period of this system is equal to natural period of vibration at viscous damping 15.11ms.

Fig. 4d represents solution of vibrations of the slab without damping, at assumption that R=0 and n=0. The natural period is equal to natural period of vibration with damping in the form of constant damping force, which is 14.95ms.

4. NUMERICAL SOLUTION OF THE MODEL

Numerical solution of equation (15) at initial conditions (33) [9], [10], [11] is presented below. The subject of analysis was a composite slab whose width was b=0.59m, height of bottom layer h_d=0.07m, height of top layer h_g=0.11m and span l=2m. Stiffness of the slab after delamination is k_{roz}=13.33MN/m and before delamination k_{mon}=46.45MN/m. Angular frequency of vibrations and parameters corresponding to viscous damping determined from formulas (16) and (32) are respectively: ω_{0roz}=225.16rad/s,

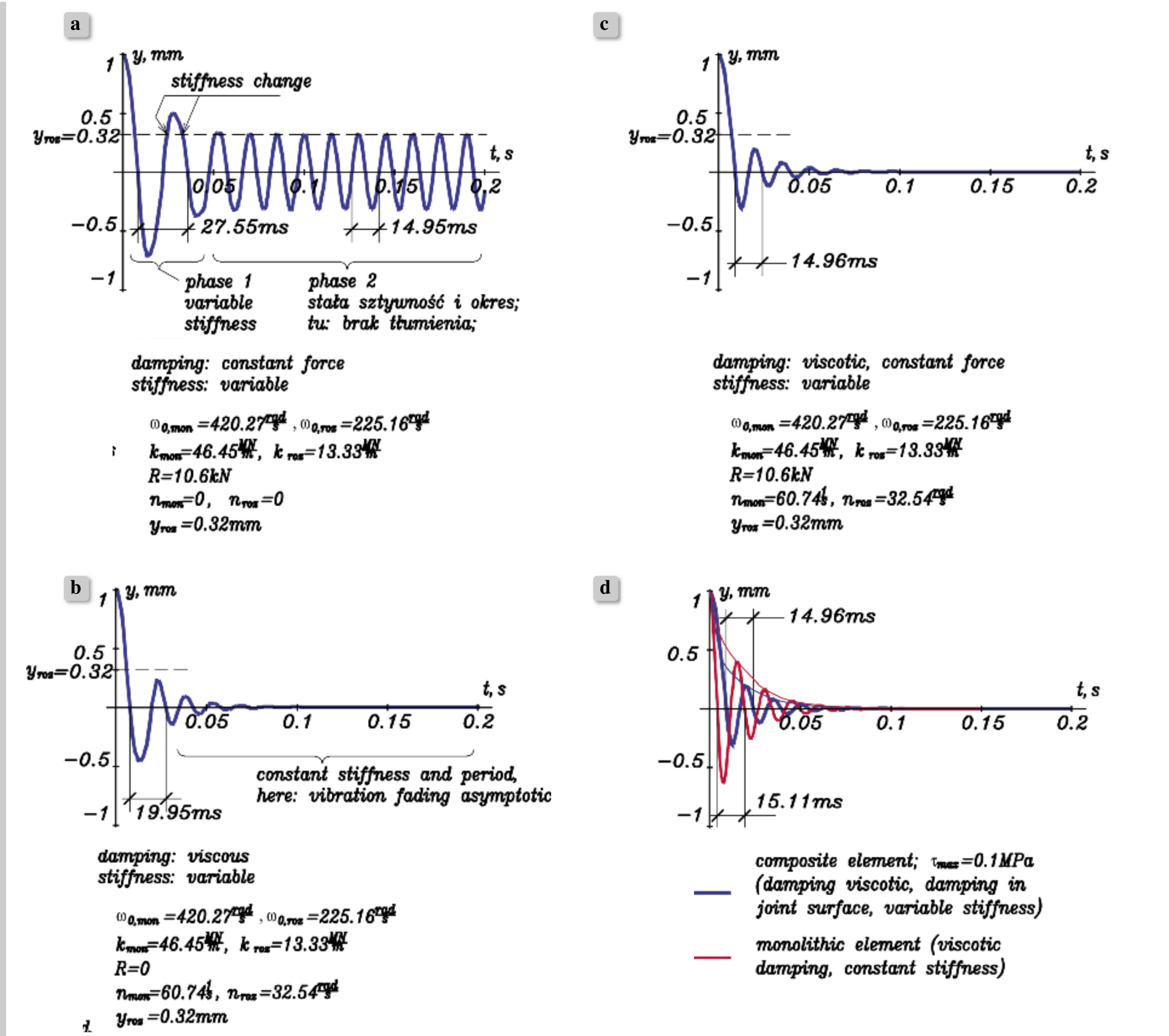

Figure 5.
Numerical solution of equation (15) with initial conditions (33) on the assumption that y_{roz}=0.32 mm, a) solution for n=0 and R≠0, b) solution for n≠0 and R=0, c) solution for n≠0 and R≠0, d) comparison of solution in Fig. 4b (vibrations of monolithic structure) with solution of the equation in Fig. 5c (vibrations of composite structure)

ω_{0mon}=420.27rad/s, n_{roz}=32.54s^{-1}, n_{mon}=60.74s^{-1}. It has to be emphasised that parameters of not delaminated slab, that is parameters with index "mon", coincide with parameters of the slab described in previous section, in which the problem was solved analytically (case of monolithic structure).

Numerical calculations were carried out for two values of maximum stresses τ_{max}=0.1MPa and τ_{max}=0.01MPa. These values correspond respectively to two maximum values of deflection: y_{roz}=0.32mm and y_{roz}=0.032mm, (determined from formula (7)), at which delamination occurs.

The calculations were carried out for three different variants of damping at each value of maximum stress τ_{max} and presented in Fig. 5 (τ_{max}=0.1MPa) and Fig. 6 (τ_{max}=0.01MPa).

The following issues were considered in those pictures:

a) vibrations with damping in the form of constant damping force and without viscous damping (Fig. 5a and 6a),

b) damping in the form of viscous damping; friction in joint surface omitted (Fig. 5b and 6b),

c) systems in which combined damping by force R and viscous damping occur (Fig. 5c and 5c). This

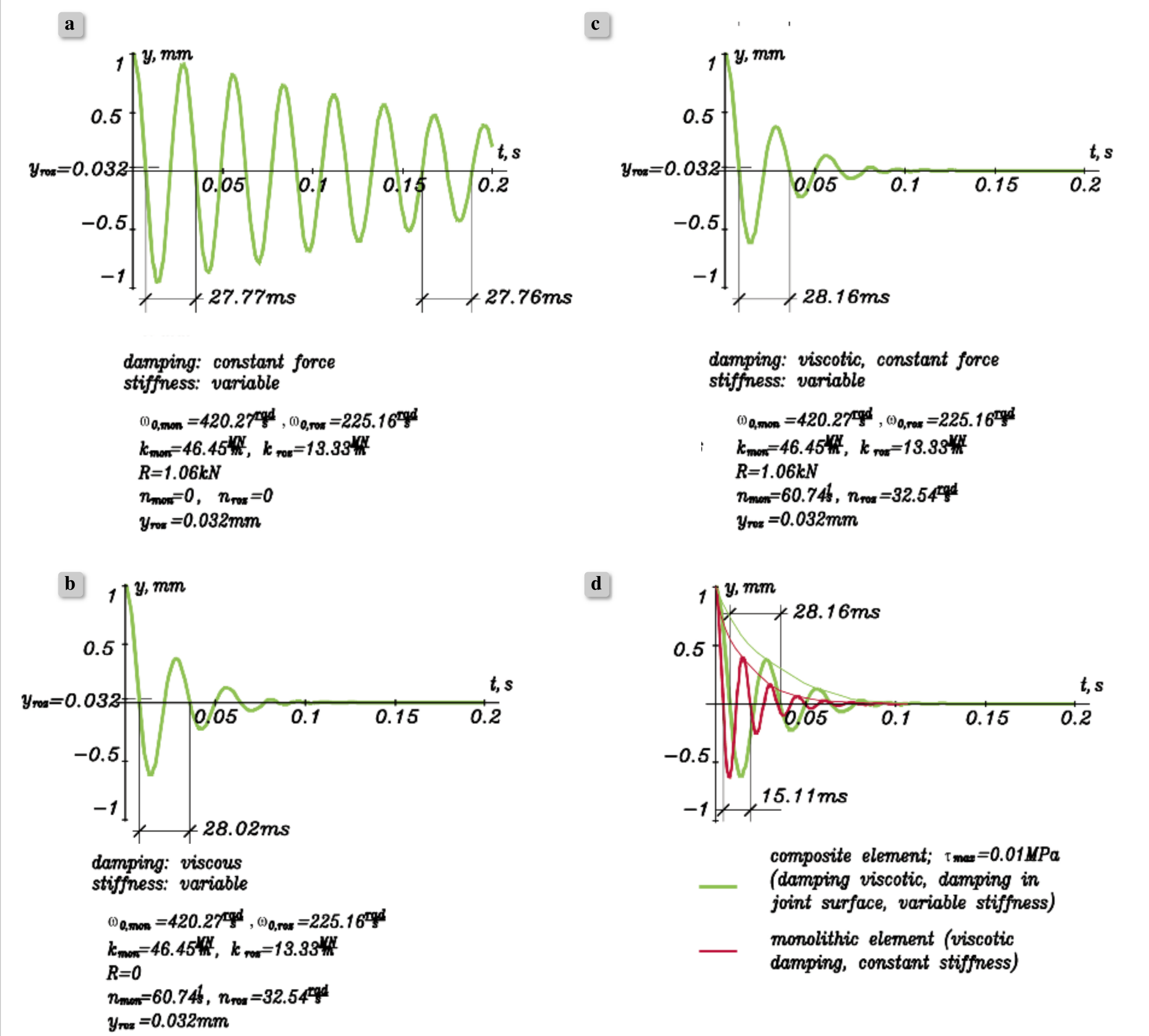

Figure 6.
Numerical solution of equation (15) with initial conditions (33) on the assumption that y_{roz}=0.032 mm, a) solution for n=0 and R≠0, b) solution for n≠0 and R=0, c) solution for n≠0 and R≠0, d) comparison of solution of equation (16) at n≠0 and R≠0 (vibrations of composite structure) with solution of equation in Fig. 4b (vibrations of monolithic structure)

case corresponds to vibrations of composite structure in which damping in joint surface occurs after delamination.

Fig. 5d and Fig. 6d compare obtained results corresponding to case c) with corresponding monolithic model, whose solution is represented in Fig. 4b.

Analysis of results a), b), c) in Figs. 5 and 6 show two phases. First phase corresponds to situation, when amplitude of vibration is greater than value y_{roz}. Second phase corresponds to situation, where amplitude of vibrations decreased below y_{roz}.

Diagrams corresponding to damping by constant force (Fig. 5a and Fig. 6a) show that in the first phase vibrations fade in a way similar to that presented in Fig. 4a. Amplitudes of vibrations cannot be limited by a straight line which is an effect of change of systems stiffness when it crosses position $\pm y_{roz}$. In consequent courses the system travels along different time periods in two ranges of stiffness k_{roz} i k_{mon}. Consequently, period of vibration of the system is not constant in this range. After limiting the amplitude of vibration to y_{roz} the system vibrates as a system without damping.

Note that the period of vibrations in Fig. 5a changed considerably. In the first phase it is around 27.55ms, omitting small changes mentioned above; in the second phase – phase of free non-damped vibrations, it is 14.95ms. The difference results from the fact that stiffness of the system corresponds to greater stiffness k_{mon} in the second phase.

In case of viscous damping, with omission of constant force damping (Fig. 5b, Fig. 6b), the vibrations fade asymptotically. Note that the period of vibrations in the first phase in Fig.5b is 19.95ms and is smaller than analogical period of vibrations in Fig. 6b, which is 28.02ms. This results from the fact that the system presented in Fig. 6b, for which y_{roz}=0.032mm, stays longer in area corresponding to stiffness k_{roz}.

Temporal courses of vibrations of systems in Fig. 5c and 6c are similar to those in Fig. 4c. This results from the fact, that with employed assumptions viscous damping has a decisive role in vibrations damping.

Fig. 5d and 6d allow to compare solutions of vibration of composite elements to solutions of monolithic elements vibration.

The comparison leads to the following conclusion. In case of assumed greater value of frictional stress (τ_{max}=0.1MPa) the fact that the joint surface exists did not considerably changed the natural period in comparison to monolithic element. However, it caused increase of damping in the first phase of vibration, where the amplitude is greater than y_{roz}. The vibration fading of composite element, where the amplitude is less than y_{roz}, and in corresponding monolithic element proceeds similarly.

In case of assumed smaller value of frictional stress (τ_{max}=0.01MPa) the vibration of the composite element in the first phase, where amplitudes of vibration are big, fade slower in comparison to monolithic element. Simultaneously the period of vibration increases considerably in comparison to monolithic element.

Comparison of lines limiting solution of vibrations in Fig. 5d and 6d is presented in Fig. 7. The comparison confirms that, depending on assumed limiting values of stress τ_{max}, different intensity of dissipation of energy per unit of time is obtained. Greater dissipation of energy per unit of time occurs when the value of stress c is greater. Then the composite structure loses more energy than monolithic structure. It has to be emphasised that the natural period of vibration of composite and monolithic structures does not differ substantially.

On the other hand, according to the model, in case of small value of stress τ_{max} the vibrations of composite structure fade slower than vibration of monolithic structure. In such situation considerable increase in period of vibration is observed in comparison to monolithic structure.

Based on the presented observations, author of the paper is conducting laboratory experiments, whose aim is to diagnose the state of composite structure on the basis of measurement of free vibrations.

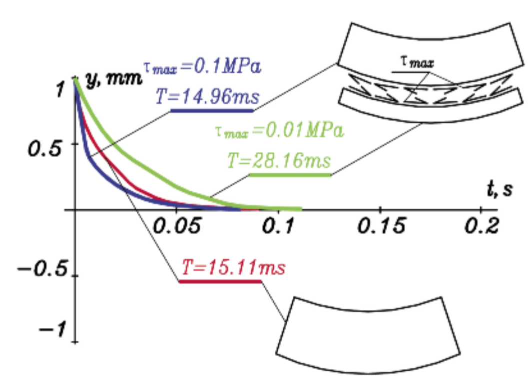

Figure 7.
Comparison of lines limiting vibrations of composite structure with viscous damping and frictional stress τ_{max}=0.1MPa and τ_{max}=0.01MPa to a line limiting vibration of monolithic structure

5. CONCLUSIONS

The paper defines model of vibrations of composite reinforced concrete structure. Damping of vibrations in joint surface is caused by frictional stress thanks to vertical reinforcement. Damping resulting from microdeformations of concrete has been modelled as viscous damping. Defined model has been solved analytically and numerically for two different values of frictional stress in joint surface. Moreover, vibrations of monolithic model (without joint surface) have been analysed. As an effect of the analysis the following conclusions may be proposed:

– greater dissipation of energy occurs when the value of frictional stress is greater, while natural periods of composite and monolithic structures do not differ substantially,
– assuming small value of frictional stress, vibrations of composite structure fade slower than vibrations of monolithic structure, the natural period of vibrations of composite structure increase in comparison to monolithic structure.

Above presented observations may serve as a basis for diagnosis of state of composite structure on the basis of measurement of free vibrations.

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