A R C H I T E C T U R E C I V I L E N G I N E E R I N G

The Silesian University of Technology



## NUMERICAL MODELS AND ANALYSES OF RC FRAME CORNERS UNDER OPENING MOMENT

**FNVIRONMENT** 

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#### Abstract

In the frame corners under opening bending moment there is a complex and non-linear distribution of stress. In such parts of structure the Bernoulli's principle is not fulfilled and the choice of a proper reinforcement is quite difficult. The authors of this paper suggest the use of Strut-and-Tie and FEM analysis as recommendable methods for the corners under opening bending moment.

In this paper a few typical reinforcement details are considered: reinforcement detail only with loop-shaped main bars, detail with one diagonal stirrup, detail with three parallel diagonal stirrups, detail with two fan-shaped stirrups and detail with three fan-shaped stirrups.

The other corner parameters are the same for all cases. The authors considered two different cases: section heights of column and beam are the same and section heights of column and beam are different, namely the beam is higher.

The first step of these analyses is to establish the required reinforcement using Strut-and-Tie method. The next step is FEM analysis performed in Abaqus software. In this step ULS and SLS are checked and yielding, history of load, cracking and non-linear behaviour of each corner detail are recreated. Thanks to corner efficiency factor, the most reasonable reinforcement detail is established.

#### Streszczenie

W ramach żelbetowych w narożach poddanych działaniu momentu otwierającego panuje złożony i nieliniowy rozkład naprężeń. W takich fragmentach konstrukcji nie jest spełniona zasada Bernoulliego i dobór właściwego zbrojenia jest utrudniony. Autorzy tego referatu proponują użycie metody Strut-and-Tie i metody elementów skończonych jako skutecznych metod analizy naroży pod działaniem momentu otwierającego.

W referacie rozważa się kilka typowych detali zbrojenia takich naroży: detal tylko z głównym zbrojeniem w kształcie pętli, detal z dodatkowym strzemieniem ukośnym, detal z trzema dodatkowymi równoległymi strzemionami ukośnymi oraz detale z dwoma i trzema strzemionami ukośnymi ułożonymi wachlarzowo. Pozostałe dane geometryczne i materiałowe są identyczne dla wszystkich detali. Autorzy rozważają ponadto dwa różne przypadki: gdy wysokości przekrojów poprzecznych belki i słupa tworzących naroże są te same oraz gdy wysokości te są różne, a ściślej wysokość przekroju belki jest większa niż przekroju słupa.

Pierwszym krokiem tych analiz jest ustalenie wymaganego pola przekroju poprzecznego zbrojenia używając metody Strut-and-Tie. Następnym krokiem jest analiza w MES przeprowadzona w programie Abaqus. W tym drugim kroku sprawdzane są stany graniczne nośności oraz użytkowania, a także odtworzona jest historia obciążenia, zarysowanie oraz nieliniowe zachowanie naroża dla każdego z detali zbrojenia. Dzięki określeniu współczynnika efektywności naroża zostaje dobrany najbardziej racjonalny detal zbrojenia.

Keywords: Abaqus; Concrete damaged plasticity; Frame corners; Opening bending moment; Strut-and-Tie Method.

## **1. INTRODUCTION**

Frame corners are one of the most difficult parts of concrete structures, especially when opening bending moment occurs. In such corners the Bernoulli's principle is not fulfilled and stress distribution is non-linear and therefore traditional methods of determining a proper reinforcement are not advisable. These regions are often called D-regions (D for disturbance) in contrast to B-regions (B for Bernoulli). For many years structure designers were applying the reinforcement of such corners as based on handbook recommendations or intuitively. In some cases this procedure could lead to a thriftless use of reinforcement steel, problems during concrete placing and mixing on the construction site or even too large crack propagation.

One of the simplest and most transparent method of analysis of the corners is Strut-and-Tie Method. Some important advice on use of this method for corners is included in Eurocode [1], where both closing and opening bending moments are considered. However, for the opening bending moment only one situation is described, namely when the section heights of beam and column are the same. There are no recommendations for different heights, likewise there is a lack of laboratory research taking into account different section heights. The authors of this paper try to complement this situation and recommend the reasonable reinforcement of corners and general rules of analysis using both FEM and Strutand-Tie Method.

## 2. ASSUMPTIONS

For purpose of Strut-and-Tie and FEM analyses some general assumptions have been done. Two different cases are taken into account: when sections heights are the same and when they are different. Geometry of corners for both cases is presented in Figs. 1 and 2.



Geometry of corner in case of the same section heights



Geometry of corner in case of different section heights

All analyzed corners are calculated as made of concrete C40/50 and reinforcement steel B500SP. The material constants are:

- concrete:  $f_{ck} = 40$  MPa,  $f_{cd} = 34.30$  MPa,  $E_{cm} = 35$  GPa,  $\nu = 0.167$ ,
- steel:  $f_{yk} = 500 \text{ MPa}, \qquad f_{yd} = 434.8 \text{ MPa},$  $E_s = 200 \text{ GPa}, \nu = 0.3.$

The load of each corner is an opening bending moment M = 30 kNm, modelled with a pair of forces, whose magnitudes are:

- 250 kN for the section height 200 mm and the distance between the forces is 120 mm,
- 71.43 kN for the section height 500 mm and the distance between the forces is 420 mm.

The compressive behaviour of concrete is modelled with the relation presented in the Table 1. The behaviour of steel is represented in Fig. 3. For the purpose of analyses in Abaqus software, the "concrete damaged plasticity model" and the "classical metal plasticity model" are assumed [2]. Both of these models are standard models in Abaqus code.

Table 1.         Compressive behaviour of concrete		
Yield stress [MPa]	Inelastic strain [%0]	
13.70	0.00	
29.30	0.95	
34.30	1.75	
33.70	2.25	
28.00	2.95	



The tensile behaviour of concrete is established according to the fracture energy,  $G_f = 130$  N/m. Maximum inelastic strain of concrete depends on a dimension of a single finite element in mesh applied in FEM. The maximum value of strain is in the range 3.5 to 15%.

The reinforcement of all corners is established in Strut-and-Tie Method. All the analyzed reinforcement details are gathered in Fig. 4.



## Reinforcement details

## **3. STRUT-AND-TIE METHOD RESULTS**

In this section there are gathered results gained in Strut-and-Tie Method. This method allows to consider ULS for each reinforcement detail, namely required reinforcement and stresses in struts and nodes. Moreover, the corner efficiency factor is calculated for each corner. All these results are presented in Table 2 [3, 4].

The reinforcement calculated in Strut-and-Tie Method is afterwards used in models prepared in Abaqus software.

## Table 2. Result in Strut-and-Tie Method for each reinforcement detail

# 4. RESULTS OF FEM ANALYSES

The results gained in Abaqus software are presented for each corner separately and then compared together in one graph. Moreover, each corner results are compared with results of laboratory tests executed by other scientists. For each corner equivalent plastic strain at integration points, tensile damage at integration points for concrete and yielding flags for steel are presented. These results are presented in a form of maps. Dimensions and displacements are presented in mm and stress unit is GPa. In the common graph there is presented a displacement of a chosen point for each corner versus the load ratio  $\lambda$ . The point and its displacement taken into consideration are presented in Fig. 5.



Figure 5. Direction of a nodal displacement

Reinforcement detail	Corner efficiency factor	Required and provided reinforcement
Detail 1. – without diagonal stirrups and diagonal bars	0.829	Required reinforcement: 5.75 cm <sup>2</sup> Provided reinforcement: 2 bars each 20 mm diameter
Detail 2. – with a diagonal bar	0.760	Required reinforcement: 5.75 cm <sup>2</sup> Provided reinforcement: 2 bars each 20mm diameter Required reinforcement for diagonal bars: 0.74 cm <sup>2</sup>
Detail 3. – with a diagonal stirrup	1.125	Required reinforcement: for main bars 2.88 cm <sup>2</sup> ; for stirrups: 4.06 cm <sup>2</sup> Provided main reinforcement: 2 bars each 16 mm diameter; provided stirrups diameter: 20 mm
Detail 4. – with fan-shaped diagonal stirrups	1.490	Required reinforcement: main bars 1.22 cm <sup>2</sup> ; side stirrups 2.03 cm <sup>2</sup> , centre stirrup 2.56 cm <sup>2</sup> Provided reinforcement: main bars 16mm diameter, stirrups 12 mm diameter

The provided reinforcement for each corner is defined in Abaqus software as 1D beam elements. The proper constraint of steel and concrete is ensured with use of "embedded region" option, which is a standard option in Abaqus. Concrete region is meshed with 2D elements. Each corner is defined as 2D body with a proper thickness.

### 4.1. Results for detail 1

The first analyzed detail does not contain any diagonal bars or diagonal stirrups. Only main reinforcement with or without loops is provided. Strut-and-Tie Method results for this detail suggest, that the use of it is not recommendable. Results gained in FEM confirm this statement. In Fig. 6 there is presented a tensile damage at integration points and in Fig. 7 – an equivalent plastic strain at integration points.



Tensile damage for detail 1



Equivalent plastic strain for detail 1

The tensile damage map indicates that three distinct cracks propagate – two in the reflex angle of corner and one on the diagonal of corner. The equivalent plastic strain shows regions in which concrete is crushed.

The load ratio  $\lambda$  reached 0.9 and the nodal displacement of analyzed point on the equilibrium axis is

4.0 mm. Reinforcement steel yields directly in the nodal zone. The active yield flag (value 1.0 means full yielding of steel) is presented in Fig. 8. The corner efficiency factor established during laboratory tests [5] is 0.47.



Active yield flag for detail 1

### 4.2. Results for detail 2

The second reinforcement detail contains a diagonal bar in the reflex corner. On the contrary, there is no diagonal stirrup. The results for this detail leave no doubt that the use of diagonal bar does not improve corner efficiency and therefore the use of this bar is not recommendable. Though the load ratio reached 1.03, the nodal displacement is about 9.0 mm.

The maps of strains (Figs. 9 and 10) indicate that in this corner two main cracks appear and propagate, similar to detail 1. Reinforcement steel yields also directly in nodal zone (Fig. 11). This corner detail was also tested [5] and corner efficiency factor reached 0.59.



Figure 9. Tensile damage for detail 2



Figure 10. Equivalent plastic strain for detail 2



Figure 11. Active yield flag for detail 2

### 4.3. Results for detail 3

Thanks to the use of one diagonal stirrup this corner shows much better properties under the same load conditions as prior details. The most important difference is the lack of diagonal crack in nodal zone. The crack in reflex angle still propagates, but strains shown in the Fig. 12 are much lower than for detail 1. and 2.



Figure 12. Tensile damage for detail 3

It is also important that compressive strains in concrete are also lower in nodal zone (Fig. 13) and the yield flags (Fig. 14) appear outside the corner. The load ratio reaches 0.98 and nodal displacement is about 2.0 mm. Laboratory tests [5] established corner efficiency factor for this detail as 0.83. Generally, this reinforcement detail is worth recommendation.



Equivalent plastic strain for detail 3



Figure 14. Active yield flag for detail 3

### 4.4. Results for detail 4

This reinforcement detail contains three diagonal stirrups placed in fan-shaped pattern. Strut-and-Tie Method results suggest that this detail allows to gain high efficiency factor. The FEM results confirm that in this case a diagonal crack in nodal zone does not appear (Fig. 15) and yielding of steel is also outside the corner (Fig. 17). The load ratio reaches 0.96 and nodal displacement is about 2.0 mm. Laboratory tests [6] specified corner efficiency factor for this detail as 1.13. This reinforcement detail is also worth recommendation.



Tensile damage for detail 4



Figure 16. Equivalent plastic strain for detail 4



Active yield flag for detail 4

### 4.5. Equilibrium paths for all corners

The relations between the load ratio  $\lambda$  and nodal displacement for all details are shown together in Fig. 18. Details 3. and 4. show the highest stiffness and their displacement versus the load ratio are almost the same. Detail 1. reaches the lowest load ratio. For detail 2. the path shows a very clear plateau and the displacement is very large despite of the load ratio. These paths confirm that details 3. and 4. are the best solutions for reinforcement of corners under opening bending moment.



## crack in the reflex corner propagates slower,

- diagonal stirrups allow to avoid steel yielding inside the corner,

All executed calculations allow to make following

- diagonal stirrups are worth to use; their use allows to avoid a diagonal crack in nodal zone and the

- diagonal bar does not improve the behaviour of node.
- the stiffness of corner depends on use of diagonal stirrups.

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**5. CONCLUSIONS** 

conclusions:

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## REFERENCES

- EN 1992-1-1: Eurocode 2: Design of concrete struc-[1] tures - Part 1-1: General rules and rules for buildings.
- [2] Abagus 6.12 Analysis User's Manual, 2012
- Szczecina M., Winnicki A.; Strut-and-Tie Method [3] Applied to Concrete Frame Corners under Opening Bending Moment. Proc. 10<sup>th</sup> European Conference of Young Researchers and Scientists TRANSCOM 2013, Section 7 - Civil Engineering; p.315-318
- [4] Szczecina M., Winnicki A.: Analysis of Concrete Frame Corners Under Opening Bending Moment Using Strut-and-Tie Method. Proc. Konferencja Naukowa Młodych Doktorantów i Młodych Naukowców Młodzi dla Techniki, Płock, Poland, 2013; p.83-96
- Mayfield B., Kong F.K., Bennison A.; Strength and [5] Stiffness of Lightweight Concrete Corners. ACI Journal, Vol.69, No.7, 1972; p.420-427
- Skettrup E., Strabo J., Andersen J. H., Brondum-[6] Nielsen T.; Concrete Frame Corners. ACI Journal, Vol.81, No.6, 1984; p.587-593

Equilibrium paths for corner details