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IMPORTANCE OF PROFESSOR KAZIMIERZ FLAGA RESEARCH WORKS IN THE FIELD OF THERMAL-SHRINKAGE EFECTS IN CONCRETE STRUCTURES

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Abstract

The paper discussed the issues generally termed as thermal and shrinkage effects in concrete structures. These issues are the one of the most important topics covered in the scientific and engineering work of Prof. Kazimierz Flaga and the aim of this paper is to remind and summarize his contribution in this field. Professor dealt with a wide range of topics in this field, such as a temperature function and curing of concrete at different temperatures; temperature stresses and cracking due to the hydration heat, morphology of shrinkage cracking in concrete as well as the practical prediction of possible early age thermal-shrinkage cracking in externally restraint concrete structures.

Streszczenie

W artykule omówiono zagadnienia ogólnie określane jako efekty termiczno-skurczowe w konstrukcjach betonowych. Zagadnienia te są jednym z najważniejszych tematów naukowych i inżynierskich podejmowanych przez prof. Kazimierza Flagę, a celem niniejszego artykułu jest przegląd i podsumowanie wkładu prof. K. Flagi w tę dziedzinę konstrukcji betonowych. W ramach tematyki termiczno-skurczowej Profesor zajmował się szeroką gamą zagadnień, między innymi takich jak funkcja temperatury i dojrzewanie betonu w warunkach zmiennych temperatur, naprężenia i rysy termiczne wynikające z ciepła hydratacji, morfologia rys skurczowych w betonie, a także praktyczne przewidywanie oceny ryzyka wystąpienia wczesnych rys termiczno-skurczowych w konstrukcjach o ograniczonej swobodzie odkształceń.

Keywords: **Temperature; Thermal stresses; Shrinkage stresses; Cracking; Massive concrete structures.**

1. INTRODUCTION

Professor Kazimierz Flaga is well-known and highly appreciated personality by civil engineering community. From the very beginning his professional carrier has been connected with the Cracow University of Technology. He was the Rector of the Cracow University of Technology for two terms as well as the Head of Institute of Building Materials and Structures. In 2011 Professor Kazimierz Flaga was honored with Doctor Honoris Causa distinction by the Cracow University of Technology.

Professor Kazimierz Flaga actively works in the field of scientific, educational as well as professional research. Achievements, resulting from 52 years of work are impressive – a list of scientific works of Professor includes 332 items, of which 16 are monographs and another 316 are articles and scientific papers. Simultaneously, the area of scientific interests of Professor is vast and covers a wide range of topics, from the complex issues of the construction of bridges and tunnels, through the problems of concrete technology to the general structural issues. Concrete structures are also included in the area of scientific interests of Prof. K. Flaga. The main areas of professional and scientific activities of Prof. K. Flaga in the field of concrete structures have been and remain as the following:

- basic issues of concrete and reinforced concrete mechanics;
- ultimate and serviceability limit state of reinforced concrete structures,
- properties of structural concrete,
- thermal and shrinkage stresses in concrete structures and analysis of cracking risk in early age massive concrete structures,
- minimum reinforcement due to the possible cracks and surface reinforcement in concrete structures,
- destruction of the concrete under compression,
- the use of concrete in buildings sports,
- composite concrete structures,
- design of reinforced concrete and prestressed concrete structures.

Undoubtedly, it is difficult to summarize in synthetic form in one paper all these issues, developed over many years by Professor. Only subjectively selected one topic of wide scope of scientific research and engineering works of Prof. K. Flaga is outlined in this paper. Thus, this paper is focused on issues generally described as "thermal-shrinkage effects in concrete structures" which are the most important topics covered and developed by Professor throughout the all period of scientific and engineering activity. Professor has dealt with a wide range of issues within this theme, but what is the most important he was the forerunner of these issues in Poland.

2. CURING OF CONCRETE AT DIFFERENT TEMPERATURES

The problem with high temperatures arising during the curing and hardening of concrete has been known since the 30s of last century when first dams were built in the United States. The efforts of engineers and researchers were directed mainly to the development of measurement techniques, those that would be able to control the curing temperature of concrete, strains and furthermore detect possible cracks. The first scientific works were focused on the issues of heat transfer, determination of non-stationary temperature fields in the massive concrete structures and methods for predicting the development of thermal and shrinkage stresses. The subsequent theoretical works were inspired by the construction of large dams on the rivers of Siberia in the 50s of the twentieth century. It should be also mentioned that first recommendations and guidelines on structures subjected to early age thermal and shrinkage deformations were developed scarcely in the seventies of last century.

Without any doubt it can be stated that Professor Kazimierz Flaga began national achievements in this subject. The work of Professor from the sixties [1, 2] take on the impact of elevated temperatures on the rate of the cement hydration process and the development of the mechanical properties of curing concrete. The extensive research of the temperature function determined on the basis of changes in the strength of hardening concrete are presented in the cited articles. The results of these studies confirmed the suggestions of other researchers, those who indicate that there is a certain limit of value of the temperature depending on the type of cement, above which the established functions describing the temperature dependence cannot be applied. It should also be noted that these studies were pioneering, because the concept of a function of temperature, specifying the physicochemical reaction rate under conditions of elevated temperatures was introduced only a few years earlier.

Professor continued this theme in next works [3, 4, 5], with the special attention to practical engineering applications of the conducted theoretical research. Among others he proposed an analytical method for determining of the optimal time for shuttering removal of concrete structures curing under variable temperatures. This method was based on a function of temperature modified by Professor and the equivalent age of concrete. It is also worth to mention that Prof. K. Flaga proposed a modified formula describing the development of the heat of hydration in concrete that was developed later in one of the PhD thesis carried out by him (M. Andreasik).

$$q(T,t)=0{,}5\left(0{,}75\frac{w}{c}+0{,}77\right)aQ_{\infty}\cdot 2^{9{,}8\frac{T-20}{T+78}}\cdot\left[\int_0^t 2^{9{,}8\frac{T-20}{T+78}}dt\right]^{-1{,}5}\cdot e^{\left[-a\left(0{,}75\frac{w}{c}+0{,}77\right)\left(\int_0^t 2^{9{,}8\frac{T-20}{T+78}}dt\right)^{-0{,}5}\right]}$$

where

T – curing temperature, °C,

t – age of concrete in days,

Q_∞ – total heat of hydration, J/g,

a – coefficient depending on the type of cement,

$\frac{w}{c}$ – water-cement ratio.

Next works were devoted to the analysis of temperature fields in an early age concrete and the rate of erection time of the massive concrete structures [4]. The differential formulation of the temperature development was proposed and the calculations were performed on a digital machine "Odra" that was available at this time. Already in these early works Prof. K. Flaga emphasized the importance of the early thermal and shrinkage effects in the construction of massive concrete structures and simultaneously pointed to the significant influence of technological factors on the development of discussed effects.

3. ESTIMATION OF THE EARLY AGE THERMAL CRACKING TENDENCY IN CONCRETE STRUCTURES

The hydration of cement is a highly exothermal reaction and as a result concrete elements are subjected to the temperature variations. In structural elements with thick sections the internal temperature can reach a significant level. Furthermore, the internal temperature drops slowly while the surfaces with direct contact with environment cool rapidly. As a result, thermal gradients occur across the section of concrete elements. There are also moisture gradients due to the differences in loss of moisture from the surfaces and from the core of the element. The volume changes due to the temperature and moisture variation have consequences in arising stresses in a concrete element. These stresses can be defined as self-induced stresses – they are related to internal restraints of the structure, resulting from non-uniform volume changes in a cross section. In internally restraint elements during the phase of temperature increase tensile stresses originate in surface layers of the element and compressive stresses are observed inside the element. An inversion of the stress body occurs during the cooling phase: inside we observe tensile stresses, in the surface layers – compressive stresses. The self-induced stresses can be expected, for example, within thick foundation slabs, thick walls, dams and each element with interior temperatures considerably greater than surface temperatures.

The crucial question here is: what elements are sensitive to the early age thermal and shrinkage effects and when they can be classified as a massive structure. In ACI 116R massive concrete is defined as "any volume of concrete with dimensions large enough to require that measures to be taken to cope with the generation of heat and attendant volume change to minimize cracking". Because it is not the precise definition, some measures are proposed to estimate the early age thermal cracking tendency. One of them was proposed by Prof. K. Flaga and it is related to the surface modulus defined as:

$$m=\frac{S}{V}$$

where *S* is the area of surfaces and *V* is the volume of element.

According to the above mentioned proposal given by Prof. K. Flaga, the concrete element is defined as massive or thick element that is sensitive to early age thermal effects when $m<2$. In such elements the expected maximum temperature is greater than 20°C. When $2 \le m \le 15$ the element is defined as medium-thick and the expected maximum temperature difference is lower than 20°C. It should be also mentioned that in medium-thick elements the shrinkage effects play more important role than in mass elements.

4. ENGINEERING MODEL FOR ASSESSMENT OF EARLY AGE THERMAL-SHRINKAGE STRESSES IN RC TANKS WALLS AND BRIDGE ABUTMENTS

The subject of non-uniform thermal and shrinkage volume changes of concrete and generated because of this stresses is developed by Professor to this day. Many works can be mentioned here [12,13, 18, 19, 21, 22, 24, 28, 29, 30, 31, 32, 36, 38, 39, 42], which present the complex nature of the early thermal-shrinkage stresses, experiences and methods of reducing the early age cracking risk in various concrete structures, from massive foundation slabs to the tank walls, retaining walls or bridge abutments.

The important Professor's contribution in this field is the proposal of the simplified, engineering method to estimate the concrete curing temperature of concrete and the stress arising as a result of these temperatures [37, 44]. The widespread use of this method, especially in the design of bridge abutments walls can be confirmed by the designers of bridge structures.

The analytical approach proposed by Prof. K. Flaga consists of three steps: prediction of thermal strains, shrinkage strains and thermal-shrinkage stresses.

Self-heating temperature and thermal strains

Increase of concrete temperature in elements due to self-heating in the process of cement hydration, ΔT_{heat}, depends on a number of factors. Its value is to some extent proportional to the value of self-heating in massive concrete elements in adiabatic conditions, equal to:

$$\Delta T^{adiab} = \frac{c \cdot Q_h(\tau)}{c_b \cdot \rho_b}, \tag{1}$$

where:

c – amount of cement in 1 m^3 of concrete mix, kg;

$Q_h(\tau)$ – amount of hydration heat, kJ/kg; it can be assumed that about 70% of the total heat of hydravion is released in first days of concrete curing,

c_b – specific heat of concrete, kJ/(kg·K);

ρ_b – density of concrete, kg/m^3.

This means that concrete of initial temperature T_{bo} in the interior of the element would heat up to the temperature T^{adiab} equal to:

$$T^{adiab} = T_{bo} + \Delta T^{adiab}, \tag{2}$$

In reality the self-heating temperature T_{int} is lower:

$$T_{int} = T_{bo} + \chi \Delta T^{adiab} \tag{3}$$

due to the heat exchange with environment (χ is equal 1 only for perfectly adiabatic conditions, for other cases $\chi<1$).

The value of mean self-heating in the cross section can be taken as approximately equal to:

$$T_m = T_{int} - \frac{1}{3}\left(T_{int} - T_p\right) \tag{4}$$

For more precise calculations, the value of surface temperature must be determined accounting for heat dissipation. The surface gradient of thermal fields $\left.\frac{dT(\tau)}{dx}\right|_p$ is determined based on the thermal transfer coefficient α_p (Fig. 1).

Thermal strains can be calculated based on predetermined temperature according to the equation:

$$\Delta\varepsilon_T = \alpha_T \Delta T \tag{5}$$

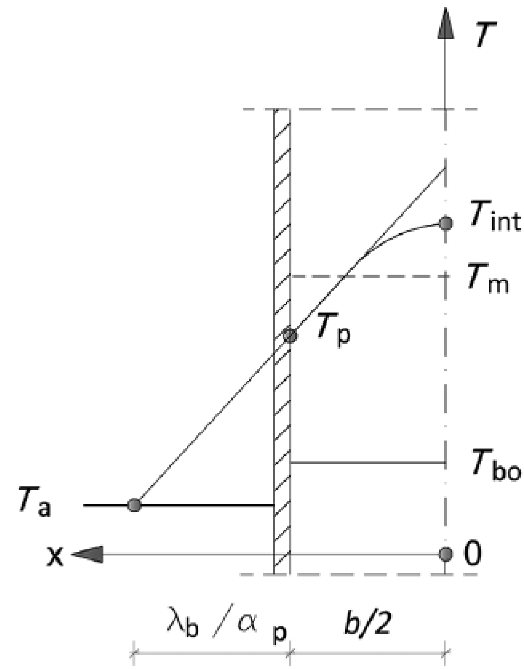

Figure 1.
Temperature distribution in the cross section of the wall

where α_T is the coefficient of thermal expansion and ΔT is equal to:

$$\Delta T = \gamma\left(T_m - T_a\right) \tag{6}$$

where T_a is ambient temperature and γ is the coefficient considering the fact that the differences in temperatures between the wall and the foundation slab is smaller because the foundation slab is also being heated by the wall.

Shrinkage strains

The shrinkage strain can be calculated according to (Eurocode 2). Total shrinkage strain ε_{cs} is a sum of two components: drying shrinkage strain ε_{cd} and autogenous shrinkage strain ε_{ca}:

$$\varepsilon_{cs} = \varepsilon_{cd} + \varepsilon_{ca}. \tag{7}$$

Development of drying shrinkage in time can be defined according to the equation:

$$\varepsilon_{cd}(t) = \beta_{ds}(t, t_s) \cdot k_h \cdot \varepsilon_{cd,0}, \tag{8}$$

where:

k_h – coefficient dependent on notional size of concrete element h_0 relating cross-section element to the perimeter in contact with atmosphere;

$\varepsilon_{cd,0}$ – free drying shrinkage strain;

$\beta_{ds}(t,t_s)$ – relationship defining the actual drying shrinkage at the moment t for a given moment of the beginning of drying process t_s, given by the formula:

$$\beta_{ds}(t,t_s) = \frac{t - t_s}{(t - t_s) + 0.04 \cdot \sqrt{h_0^3}}. \qquad (9)$$

Development of autogenous shrinkage, which is crucial in early ages of concrete curing, can be defined by the function:

$$\varepsilon_{ca}(t) = \beta_{as}(t) \cdot \varepsilon_{ca,\infty}, \qquad (10)$$

$\varepsilon_{ca,\infty}$ – final value of autogenous shrinkage strain, defined for a given class of concrete acc. to its 28-day characteristic compressive strength f_{ck} [MPa] as:

$$\varepsilon_{ca,\infty} = 2.5 \cdot (f_{ck} - 10) \cdot 10^{-6}, \qquad (11)$$

$\beta_{as}(t)$ – relationship defining the actual autogenous shrinkage at time t, acc. to the formula:

$$\beta_{as}(t) = 1 - e^{(-0.2\sqrt{t})}. \qquad (12)$$

Determination of total shrinkage strains in an older part of the structure (foundation, element I) and a newer part of the structure (wall, element II) allows to determine a strain difference resulting from different times of concrete casting of these parts, $\Delta\varepsilon_{cs}$. The strain difference is equal to:

$$\Delta\varepsilon_{cs} = \varepsilon_{cs}^{II}(t^{II}) - \left[\varepsilon_{cs}^{I}(t^{I} + t^{II}) - \varepsilon_{cs}^{I}(t^{I})\right], \qquad (13)$$

$\varepsilon_{cs}^{II}(t^{II})$ – shrinkage strain of the element II at concrete age t^{II}, days;

$\varepsilon_{cs}^{I}(t^{I}+t^{II})$ – shrinkage strain of the element I at concrete age $t^{I}+t^{II}$, days;

$\varepsilon_{cs}^{I}(t^{I})$ – shrinkage strain of the element I at concrete age t^{I}, days.

For more convenient calculations shrinkage strain difference can be expressed in a form of equivalent temperature change ΔT_{cs} as uniform cooling of the element II, according to the formula:

$$\Delta T_{cs} = \frac{\Delta\varepsilon_{cs}}{\alpha_T}, \qquad (14)$$

Thermal-shrinkage stresses

At the moment of execution of load-bearing system (a stem), the foundation is cooled down and has more or less the temperature of the surrounding air. The massive stem is in turn subjected to intensive self-heating in the process of cement hydration. At this phase it has a tendency to expand (with respect to cooler foundation). This process is accompanied with generation of small compressive forces T_1 in the concrete of the stem and tensile forces T_1 in the foundation. As soon as total bond stresses develop at the joint as a result of the bond between the old concrete and the new concrete, the concrete of the stem starts to cool down. This process, accompanied with ongoing moisture removal, leads to contraction of the wall. The contracting wall is restrained by the cooled foundation, leading to formation of compressive force T_2 in the foundation and tensile force T_2 in the joint. The bond forces T_1 and T_2 developing in the joint act on the whole cross-section of the stem, subjecting it to the eccentric load with respect to the central axis of the element (Fig. 2). The action of T_2 is especially dangerous for structural elements with a centroid located high above the joint. The forces T_2 may appear at a significant height of the stem.

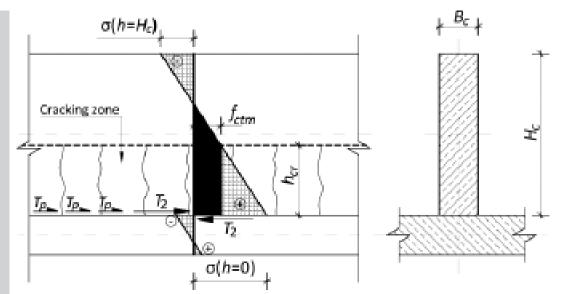

Figure 2.
Distribution of thermal-shrinkage stresses at the height of the wall cast against an old set foundation and morphology of cracking acc. to analytic, engineering approach

The simplified, analytical approach assumes that the foundation is indeformable and that forces T_2 result from the bond strength of concrete. In that manner a system is modelled in which distribution of stresses is caused by an external restraint in a form of linear restraint of thermal-moisture contraction by a stiff foundation. The bond stresses arise along the contact layer between the wall and the foundation slab and can be calculated on the base of equation:

$$\tau_p = \frac{A_c \cdot (\Delta\varepsilon_T + \Delta\varepsilon_{cs}) \cdot E_{cm,eff}(t)}{0.5 \cdot l_z \cdot b}, \qquad (15)$$

where A_c is the cross-section of the stem and $E_{cm,eff}$ is the modulus of elasticity of concrete, ε_t is the total thermal-shrinkage strains, l_z is equal to the half length of wall, b is the thickness of the wall. Simultaneously, the bond stresses predicted with the use of the equation (15) cannot exceed the maximum value of bond stress, based on the Mörsch formula as:

$$\bar{\tau}_p = 0.5 \cdot \sqrt{f_{cm} \cdot f_{ctm}}\,, \tag{16}$$

The bond force at the joint can be determined as:

$$T_2 = 0,5 \cdot \tau_p \cdot l_z \cdot b\,, \tag{17}$$

The values of stresses at the height of the wall (Fig. 4) are then equal to:

$$\sigma(h=0) = \frac{T_2}{A_c} + \frac{T_2 \cdot y_{cen}}{W_{xb}}\,, \tag{18}$$

$$\sigma(h=H_c) = \frac{T_2}{A_c} - \frac{T_2 \cdot y_{cen}}{W_{xt}}\,, \tag{19}$$

where y_{cen} is a location of the centroid of the section above the joint and W_{xb}, W_{xt} are bending indices of the section with respect to bottom (h = 0) and top (h = H_c) fibres of the stem, respectively. The expected height of the crack can be then determined from the equation:

$$h_{crack} = y_{cen} + \left(\frac{T_2}{A_c} - f_{ctm}\right) \cdot \frac{y_{cen}}{\left(\frac{T_2 \cdot y_{cen}}{W_{xb}}\right)}\,. \tag{20}$$

The stress analysis may be performed at any time of concrete curing and the values of material parameters must be defined for the actual age of concrete corrected for temperature effects and influence of creep. The change of material parameters can be taken according to (Eurocode 2). The mean modulus of elasticity development is given by the equation:

$$E_{cm}(t) = \left[\frac{f_{cm}(t)}{f_{cm}}\right]^{0.3} \cdot E_{cm}\,, \tag{21}$$

where:

E_{cm} – mean 28-day modulus of elasticity of concrete, GPa;

f_{cm} – mean 28-day compressive strength of concrete, MPa;

$f_{cm}(t)$ – mean compressive strength of concrete taking into consideration concrete age, MPa, acc. to the equation:

$$f_{cm}(t) = \beta_{cc}(t) \cdot f_{cm}\,, \tag{22}$$

in which:

$\beta_{cc}(t)$ – coefficient dependent on the age of concrete, given by the equation:

$$\beta_{cc}(t) = e^{s \cdot \left[1 - \sqrt{\frac{28}{t}}\right]}\,, \tag{23}$$

where:

t – age of concrete, days;

s – coefficient dependent on the type of cement.

Development of tensile strength is defined as:

$$f_{ctm}(t) = [\beta_{cc}(t)]^{\alpha} \cdot f_{ctm}\,, \tag{24}$$

where:

f_{ctm} – mean 28-day tensile strength of concrete, MPa;

α – coefficient, $\alpha = \begin{cases} 1 & \text{for } t < 28 \text{ days} \\ 2/3 & \text{for } t \geq 28 \text{ days} \end{cases}$.

The influence of increased temperatures due to cement hydration on development of mechanical parameters can be considered by introducing as time t the equivalent age of concrete t_e, determined according to (Model Code 2010).

The influence of creep may be considered by reduction of modulus of elasticity of concrete:

$$E_{cm,eff}(t) = \frac{E_{cm}(t)}{1 + \beta_1 \cdot \varphi_p(t_r, t_0)}\,, \tag{25}$$

where:

β_1 – ageing coefficient, can be taken as 0.8;

$\varphi_p(t_r, t_0)$ – creep coefficient, can be taken as 0.6.

5. SHRINKAGE STRAINS AND MINIMAL AREA OF REINFORCEMENT

The issues related to the determination of shrinkage strains in concrete structures [6, 7, 9, 11, 14, 21, 22, 26, 34, 36] also match to the thermal-shrinkage scope of Jubilarian works. The non-linear and non-stationary humidity fields, shrinkage strains and stresses are discussed in the cited articles. Some comments to the newly introduced standards can be also found [37]. The recommendations for the calculation of the depth of the tensile zone of concrete member given by the Professor are particularly important in this filed. It should be mentioned here that in structures with significant dimension of cross-section non-uniform humidity fields can generate tensile stress at the surface area, while the interior is subjected to compression. Determination of the depth of this tensile zone, which arises as a result of non-uniform shrinkage in concrete element is stretched, is very difficult. Prof. Flaga in his works indicated that the discussed depth depends on the degree of reinforcement, the physical features of the concrete mix, conditions of concrete curing and the massiveness of the element. Professor also proposed to calculate the depth of the tension zone of concrete as:

$b_1 \approx 0.185d$ for the medium-thick structures ($m = 2\div15$) (26)

$b_1 \approx (0.05\div0.15)d$ for the massive structures ($m<2$) (27)

where:

b_1 – depth of the tensile area;

d – thickness of the concrete element;

m – surface modulus of the concrete element.

The works related to the importance of surface reinforcement in concrete element [8, 10, 12, 15, 16, 20, 23, 25, 27, 33, 34, 36, 41] were the direct consequence of experimental and theoretical aspects of thermal shrinkage issues. These works present a thorough look at the role of surface reinforcement in concrete structures. We find in them, among other things, the analysis of the morphology of shrinkage cracks in the surface areas of free and restraint concrete elements can be found in these works. Notable effect of these works is the procedure for estimation of the amount of the surface reinforcement due to the cracking. It must be said, that also this part of the Jubilarian works is directly applied in engineering practice.

6. SUMMARY

There is no need to prove that the research activity of Professor K. Flaga significantly influenced the development of scientific and technical subjects in concrete structures. This paper only synthetically present the most important achievements of Jubilarian, selected in a subjective manner.

One important feature of the research works of Prof. K. Flaga should be also mentioned. He is always involved in innovative and timely topics in the building science and simultaneously he successfully combines the research and engineering problems. His scientific works have always been the answer – and so on – to the real problems of design and construction in the building industry. Professor's activity involving the popularization of research results and care for their suitability for the engineering practice is invaluable.

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