A R C H I T E C T U R E C I V I L E N G I N E E R I N G E N V I R O N M E N T
The Silesian University of Technology No. 2/2012

NUMERICAL STUDIES ON SIZE EFFECTS IN CONCRETE BEAMS

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Received: 10.03.2012; Revised: 30.04.2012; Accepted: 15.06.2012

Abstract

The numerical FE investigations of a deterministic and stochastic size effect in unnotched concrete beams of similar geometry under three point bending were performed within elasto-plasticity with non-local softening. Deterministic calculations were performed with the uniform distribution of a tensile strength. In turn, in stochastic calculations, the tensile strength took the form of spatially correlated random fields described by a truncated Gaussian distribution. In order to reduce the number of stochastic realizations without losing the calculation correctness, Latin hypercube sampling was applied. The numerical outcomes were compared with the size effect laws by Bazant.

Streszczenie

Przeprowadzono analizę numeryczną MES deterministycznego i statystycznego efektu skali w belkach betonowych geometrycznie podobnych poddanych 3-punktowemu zginaniu. Zastosowano sprężysto-plastyczny model betonu z nielokalnym osłabieniem. Obliczenia deterministyczne wykonano z jednorodnym rozkładem wytrzymałości na rozciąganie. Symulacje stochastyczne przeprowadzono z wykorzystaniem przestrzennie skorelowanych pól losowych opisanych obciętym rozkładem Gaussa. W celu zredukowania liczby symulacji stochastycznych, przy zachowaniu poprawności obliczeniowej, zastosowano metodę próbkowania typu sześcianu łacińskiego. Wyniki numeryczne zostały porównane z prawem efektu skali wg Bazanta.

Keywords: **Concrete beam; Elasto-plasticity; Latin hypercube sampling; Non-local softening; Random fields; Size effect; Strain localization.**

1. INTRODUCTION

A size effect phenomenon (nominal strength varies with a characteristic size of a structural member) is an inherent property of the behaviour of many engineering materials. In case of concrete materials, both the nominal structural strength and material brittleness (ratio between the energy consumed during the loading process after and before the stress-strain peak) always decrease with increasing element size under tension [1]-[3]. The results from laboratory tests which are scaled versions of the actual structures cannot be directly transferred to them.

Two size effects are of a major importance in quasi-brittle and brittle materials: energetic (or deterministic) and statistical (or stochastic) one (the remaining size effects are [3]: boundary layer effect, diffusion phenomena, hydration heat or phenomena associated with chemical reactions and fractal nature of crack surfaces). According to Bazant and Planas [3] and Bazant [4] the deterministic size effect is caused by the formation of a region of intense strain localization with a certain volume (micro-crack region – called also fracture process zone FPZ) which precedes macro-cracks. The nominal structural strength which is sensitive to the size of FPZ cannot be appropriately estimated in laboratory tests, since it differs for various specimen sizes. Strain localization volume is not negligible to the cross-section dimensions and is large enough to cause significant stress redistribution in the structure and associated energy release. The specimen strength increases with increasing ratio l_c/D (l_c – characteristic length of the micro-structure influencing both the size and spacing of localized zones, D – characteristic structure size). In turn, a statistical (stochastic) effect is caused by the spatial variability/randomness of the local material strength. The first statistical theory was introduced by Weibull [5] (called also the weakest link theory) which postulates that a structure is as strong as its weakest component. The structure fails when its strength is exceeded in the weakest spot, since stress redistribution is not considered. The Weibull's size effect model is a power law and is of particular importance for large structures that fail as soon as a macroscopic fracture initiates in one small material element. It is not however, able to account for a spatial correlation between local material properties, it does not include any characteristic length of micro-structure (i.e. it ignores a deterministic size effect) and it underestimates the effect of small- and intermediate-sizes. Combining the energetic theory with the Weibull statistical theory, a general energetic-statistical theory was developed [6]. The deterministic size effect was obtained for not too large structures and the Weibull statistical size effect was obtained as the asymptotic limit for very large structures. In turn, according to Carpinteri et al. [7], the size effect is caused by the multi-fractality of a fracture surface only which increases with a spreading disorder of the material in large structures (stress redistribution and energy release during strain localization and cracking are not considered).

In spite of many experiments exhibiting the noticed size effect in concrete and reinforced concrete elements under different loading types [8]-[17], the scientifically (physically) based size effect is not taken into account in a practical design of engineering structures, that may contribute to their failure [3], [17]. Instead, a purely empirical approach is sometimes considered in building codes which is doomed to yield an incorrect formula since physical foundations are lacking.

The goal of our numerical simulations is to determine in numbers a combined deterministic and statistical size effect in flexural resistance of unnotched beams of a similar geometry under quasi-static three-point bending by using a stochastic enhanced continuum concrete model and to compare results with existing size effect laws by Bazant [3], [4]. A finite element method with an elasto-plastic constitutive model using a Rankine'a criterion with non-local softening was used which is suitable to capture strain localization under tension. Two-dimensional calculations were performed with four different concrete beam sizes of a similar geometry. Deterministic calculations were performed assuming a constant value of the tensile strength. In turn, statistical analyses were carried out with spatially correlated random fields reflecting the random nature of a local tensile strength. The probability distribution of the tensile strength was described by a truncated Gaussian function. Random fields were generated using a conditional rejection method [18] for correlated random fields. The approximated results were obtained using a Latin hypercube sampling method [19], [20] belonging to a group of variance reduced Monte Carlo methods. This approach enables a significant reduction of the sample number without losing the accuracy of calculations.

2. SIZE EFFECT LAWS

Two size effects laws proposed by Bazant [3], [4] (called Size Effect Laws SEL) for geometrically similar structures allow for determining their nominal strength by taking into account the size-scale effect. There exist three different types of a deterministic SEL distinguished by Bazant. Type I (Fig.1a) applies to structures of a positive geometry having no notches or pre-existing cracks for which the maximum load occurs as soon as the FPZ is fully developed and the macroscopic crack can initiate. Type II (Fig.1b) occurs also for structure with a positive geometry but with notches or large stress-free cracks that grow in a stable manner up to the maximum load. Type III happens in structures of an initially negative geometry where a macro-crack can propagate up to the maximum load (it is very similar to Type II). The requirement of a positive geometry enables to incorporate the weakest link theory by Weibull. The structures obeying the size effect of Type I are sensitive to the material randomness. The nominal strength is strongly affected by the material heterogeneity and decreases with increasing structure characteristic dimension. For Type II, the effect of the material randomness can be ignored. The deterministic size effect of Type I and Type II assumes that the material strength is bound for small sizes by a plasticity limit whereas for large sizes the material follows linear elastic fracture mechanics. The following general analytical formulae for a deterministic size effect predicted by asymptotic matching were proposed by Bazant [3]

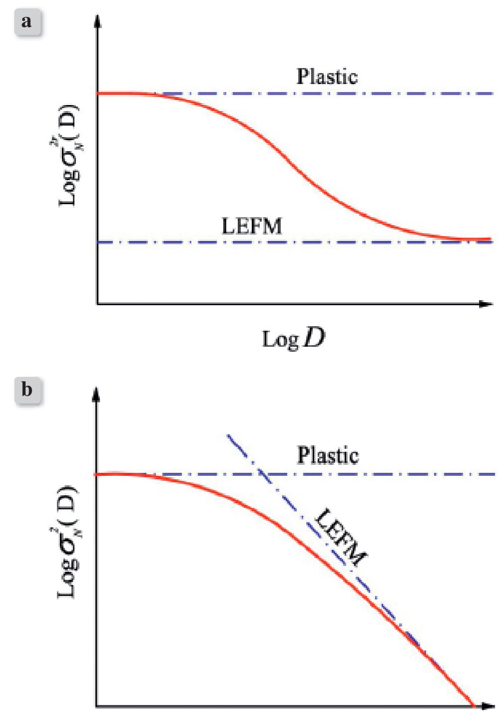

Figure 1.
Size effect laws by Bazant [4] in logarithmic scale with σ_n – nominal strength, D – characteristic structure size: a) type 1 (structures without notches and pre-existing large cracks), b) type 2 (notched structures) (material strength is bound for small sizes by plasticity limit whereas for large sizes, the material follows Linear Elastic Fracture Mechanics LEFM)

$$\sigma_N(D) = f_r^{\infty}\left(1+\frac{rD_b}{l_p+D}\right)^{\frac{1}{r}} \text{ (Type 1 size effect law SEL)} \quad (1)$$

and

$$\sigma_N(D) = \frac{Bf_t}{\sqrt{1+\dfrac{D}{D_o}}} \text{ (Type 2 size effect law SEL),} \quad (2)$$

where σ_n is the nominal strength, D is the characteristic structure size, l_p is the second deterministic characteristic length, f_r^{∞}, D_b and r denote the positive constants representing unknown empirical parameters to be determined; f_r^{∞} represents the solution of the elastic-brittle strength reached as the nominal strength for large structures, r controls the curvature and shape of the law and D_b is the deterministic characteristic length having the meaning of the thickness of the cracked layer (if $D_b=0$, the behaviour is elastic-brittle, Eq. 1). In turn, in Eq. 2, the parameter f_t denotes the tensile strength, B is the dimensionless geometry-dependent parameter (depending on the geometry of the structure and crack) and D_o denotes the size-dependent parameter called transitional size (both unknown parameters to be determined). The formula represents the full size range transition from the perfectly plastic behaviour (for $D \to 0$, $D \le l_p$) to the elastic brittle behaviour (for $D \to \infty$, $D >> D_b$) through the quasi-brittle one. The second deterministic characteristic length l_p governs the transition to plasticity for small sizes D. The case $l_p \neq 0$ shows the plastic limit for vanishing size D. This case is asymptotically equivalent to the case of $l_p=0$ for large D. The asymptotic prediction for small and large sizes leads to

$$\lim_{D\to 0} \sigma_N(D) = f_r^{\infty}(1 + rD_b / l_p)$$
$$\text{And} \qquad \lim_{D\to\infty} \sigma_N(D) = f_r^{\infty}. \quad (3)$$

The length l_p equals

$$l_p = \frac{rD_b}{\eta_p - 1} \quad (4)$$

with η_p – the ratio between the maximum plastic and elastic strength.

In turn, a formula for a coupled deterministic-stochastic size effect law involves both a deterministic scaling length D_b and a stochastic scaling length L_o [21]

$$\sigma_N(D) = f_r^{\infty}\left(\left(\frac{L_o}{D+L_o}\right)^{\frac{r\times n}{m}} + \frac{rD_b}{D+l_p}\right)^{\frac{1}{r}}, \quad (5)$$

where m is the dimensionless Weibull modulus (shape parameter of Weibull distribution) responsible for the slope of a large-size asymptote and n is the number of spatial dimensions in which the structure is scaled ($n=2$). Thus, the mean size effect is separately divided into a stochastic part and deterministic one. The parameter D_b drives the transition from elastic-brittle to quasi-brittle and L_o drives it from constant property to local Weibull via strength random field. The simplest choice for analyses is usually $L_o=D_b$. Equation 5 satisfies three asymptotic conditions: a) for small structures $D \to 0$, it asymptotically reaches the plastic limit (Eq. 3), b) for large sizes $D \to \infty$, it asymptotically reaches the dominating Weibull size effect with the slope equal to $-n/m$ ($\lim_{D\to\infty} \sigma_N \propto D^{(-n/m)}$) and c) for $m \to \infty$ and $L_o \to \infty$, it is equal to the deterministic size effect law. Thus, Eq. 5 can be regarded as the asymptotic matching of small-size deterministic and large-size stochastic size effects. The modulus m can be calculated from the coefficient of variation *cov*

$$cov = \sqrt{\frac{\Gamma\left(1+\frac{2}{m}\right)}{\Gamma^2\left(1+\frac{1}{m}\right)}-1} \qquad (6)$$

3. CONSTITUTIVE ELASTO-PLASTIC MODEL WITH NON-LOCAL SOFTENING

To describe the behaviour of concrete under tension during three-point bending, a Rankine criterion was used, for which the yield function f with isotropic softening defined as

$$f = \max\{\sigma_1, \sigma_2, \sigma_3\} - \sigma_t(\kappa), \qquad (7)$$

where: σ_i, $i=1, 2, 3$, are the principal stresses, σ_t is the tensile yield stress and κ denotes the softening parameter equal to the maximum principal plastic strain ε_{1p}. The associated flow rule was assumed. To model the concrete softening under tension, the exponential curve by Hordijk [22] was chosen:

$$\sigma_t(\kappa) = f_t\left[\left(1+(A_1\kappa)^3\right)\exp(-A_2\kappa) - A_3\kappa\right], \qquad (8)$$

where f_t stands for the tensile strength of the concrete (f_t=3.6 MPa). The constants A_1, A_2 and A_3 are assumed in the form

$$A_1 = \frac{c_1}{\kappa_u}, \quad A_2 = \frac{c_2}{\kappa_u}, \quad A_3 = \frac{1}{\kappa_u}\left(1+c_1^3\right)\exp(-c_2), \qquad (9)$$

wherein κ_u=0.005 denotes the ultimate value of the softening parameter, and the constants c_i are c_1=3 and c_2=6.93. The modulus of elasticity was assumed to be E=38.5 GPa and the Poisson ratio was υ=0.24. The tensile fracture energy was $G_f=g_f \times w_c$=52 N/m (g_f – area under the softening function, w_f=15 mm – width of the localized zone with l_c=5 mm).

To properly describe strain localization, to preserve the well-posedness of the boundary value problem, to obtain mesh-independent results and to include a characteristic length of micro-structure for simulations of a deterministic size effect, a non-local theory was used as a regularization technique [23], [24]. In the calculations, the softening parameters κ were assumed to be non-local ($\bar{\kappa}$) (according to Brinkgreve [25])

$$\bar{\kappa}(\boldsymbol{x}) = (1-m)\kappa(\boldsymbol{x}) + m\frac{\int_V \omega(\|\boldsymbol{x}-\xi\|)\kappa(\xi)\,\mathrm{d}\xi}{\int_V \omega(\|\boldsymbol{x}-\xi\|)\,\mathrm{d}\xi}, \qquad (10)$$

where $\bar{\kappa}$ (x) is the non-local softening parameter, V – the volume of the body, x – the coordinates of the considered (actual) point, ξ – the coordinates of the surrounding points in a certain neighborhood of the considered point and ω – the weighting function. The non-locality parameter m controls the size of the localized plastic zone and the distribution of the plastic strain. For m=0, a local approach is obtained and for m=1, a classical non-local model is recovered. If the parameter m>1, the influence of non-locality increases and the localized plastic region reaches a finite mesh-independent size. The softening non-local parameters $\bar{\kappa}$ near boundaries were calculated also on the basis of Eq. 10 (which satisfies the normalizing condition). As a weighting function ω, a Gauss distribution function was used

$$\omega(r) = \frac{1}{l_c\sqrt{\pi}}e^{-\left(\frac{r}{l_c}\right)^2}, \qquad (11)$$

where r is a distance between two material points. The averaging in Eq.10 is restricted to a small representative area around each material point (the influence of points at the distance of $r=3l_c$ is only of 0.01%). A characteristic length is usually related to the micro-structure of the material (e.g. maximum aggregate size). It is determined with an inverse identification process of experimental data. However, the determination of the representative characteristic length of micro-structure l_c is very complex in concrete since strain localization can include a mixed mode (cracks, shear zones) and the characteristic length (which is a scalar value) is related to the fracture process zone with a certain volume.

The calculations were carried out with a characteristic length of l_c=5 mm and m=2 on the basis of our model experiments with reinforced concrete beams failing by shear using a digital image correlation (DIC) technique [26] and earlier FE calculations with the same constitutive model [27], [28]. The non-local model was implemented in the commercial finite element code ABAQUS [29] with the aid of subroutine UMAT (user constitutive law definition) and UEL (user element definition) for efficient computations [30].

4. FE-INPUT DATA

The two-dimensional FE-analysis of free-supported unnotched beams was mainly performed with 4 different beam sizes of a similar geometry $D{\times}L_t$: 8×32 cm^2 (called small-size beam), 16×64 cm^2 (called medium-size beam), 32×128 cm^2 (called large-size beam), 192×768 cm^2 (called very large-size beam) (D – beam height, L_t – beam length), Fig. 2. The span length L was equal to 3D for all beams. The depth of the specimens was $t=$ 4 cm. The size $D{\times}L_t{\times}t$ of the first 3 beams was similar as in the corresponding experiments carried out by Le Bellego et al. [13] and Skarżyński et al. [31]. The quadrilateral elements divided into triangular elements were used to avoid volumetric locking. Totally, 13'820 (small-size beam), 39'900 (medium-size beam), 104'780 (large-size beam) and 521'276 (very large-size beam) triangular elements were used, respectively The computation time varied between 3 hours (small-size beam) and 3 days (very large beam) using PC 3.2 MHz.

In deterministic calculations, all specimens had the constant uniformly distributed tensile strength $f_t = 3.6$ MPa. In order to properly capture strain localization in concrete, the mesh was very fine in the mid-part of the beam (where the element size was not greater than 3×l_c). A quasi-static deformation of a small and medium beam was imposed through a constant vertical displacement increment Δu prescribed at the upper mid-point of the beam top.

Correlated random fields describing a fluctuation of the tensile strength were used to capture a stochastic size effect. The distribution of this single random variable f_t took the form of a truncated Gaussian function with the mean concrete tensile strength of 3.6 MPa, Fig. 3. Additionally, it was assumed that the concrete tensile strength values changed between 1.6 MPa and 5.6 MPa (f_t=3.6±2.0 MPa). To fulfil this condition, the standard deviation s_{ft}=0.424 MPa was used in the calculations. The coefficient of variations describing the field scattering was $cov=s_{ft}/\bar{f}_t$=0.118 ($\bar{f}_t$ – the mean tensile strength). The cut of variables did not visibly change a theoretical Gauss distribution since 5× s_{ft}=5×0.424=2.12 MPa. Other stochastic FE calculations showed that the assumption of a non-symmetric distribution of material parameters did not significantly affect the results [32].

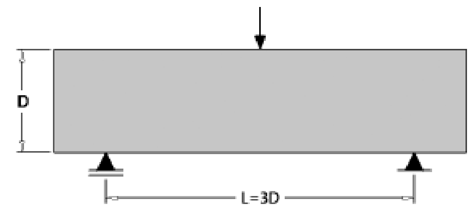

Figure 2.
Geometry of free-supported unnotched concrete beams subjected to three-point bending

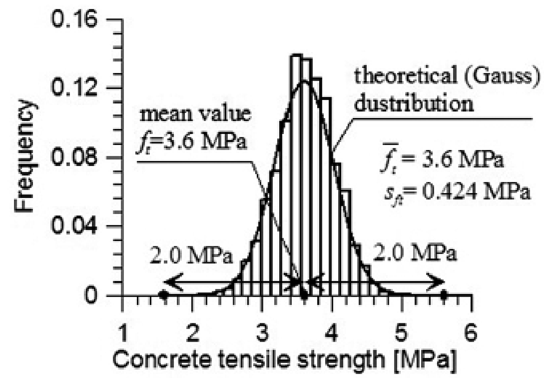

Figure 3.
Distribution of concrete tensile strength in single point of mesh

The second order homogeneous correlation function was adopted to capture the fluctuation of the tensile strength [33]

$$K(\Delta x_1,\Delta x_2)=s_{f_t}^2\times e^{-\lambda_{x_1}\Delta x_1}(1+\lambda_{x_1}\Delta x_1)e^{-\lambda_{x_2}\Delta x_2}(1+\lambda_{x_2}\Delta x_2), \tag{12}$$

where Δx_1 and Δx_2 are the distances between two field points along the horizontal axis x_1 and vertical axis x_2 and λx_1 and λx_2 are the decay coefficients (damping parameters) characterizing a spatial variability of the specimen properties (i.e. describe the correlation between the random field points). The variances and covariances were obtained based on the procedure of local averages of the random fields [34]. We took mainly into account a stronger correlation of the tensile strength f_t in a horizontal direction λ_{x1}=1.0 1/m and a weaker correlation in a vertical directions λx_2=3.0 1/m in Eq. 12 (due to the way of the specimen preparation by means of layer-by-layer from the same concrete block). The range of significant correlation was approximately 80 mm in a horizontal direction (equal to the height of a small-size beam) and 30 mm in a vertical direction (equal to the layer height formed during concrete placing). Some calculations were also carried out with a correlation range equal to 15 mm and 150 mm in a horizontal direction. The smaller the decay parameter λ is, the shorter becomes the correlation range. The dimension of the random field was identical as the finite element mesh. The same random values were assumed in 4 neighbouring triangular elements. To generate the random fields, the conditional-rejection method was used [18]. The method makes it possible to simulate any homogeneous or non-homogeneous truncated Gaussian random field described on regular or irregular spatial meshes. An important role in the calculations was played by the propagation base scheme covering sequentially the mesh points and the random field envelope which allowed for fulfilling the geometric and boundary conditions of the structure of the model. Random fields of practically unlimited sizes could be generated.

In the paper the Monte Carlo method was used which does not impose any restriction to random problems. Its only limitation is the time of calculations. A further decrease of sample numbers can be obtained using Monte Carlo variance reduction methods. The stochastic calculations according to the proposed version of the Latin sampling method were performed in two steps [35]. First, an initial set of random samples was generated in the same way as in the case of a direct Monte Carlo method. Next, the generated samples were classified and arranged in increasing order according to the chosen parameters (i.e. their mean values and the gap between the lowest and the highest values of the fields). From each subset defined in this way, only one sample was chosen for the analysis. The selection was performed in agreement with the theoretical background of the Latin sampling method. The numerical calculations were performed only for these samples. It was proved that using the Latin sampling variance reduction method the results can be properly estimated by several realizations only (e.g. 12 [35]).

5. FE RESULTS

5.1. Deterministic size effect

The evolution of the normalized flexural (horizontal normal) stress $1.5FL/(f_tD^2t)$ versus the normalized deflection u/D during three-point bending for four different beam sizes with the constant values of tensile strength f_t is shown in Fig. 4. The calculated maximum deterministic vertical forces were: F_{max}=3.83 kN (D=8 cm), F_{max}=6.75 kN (D=16 cm), F_{max}=12.57 kN (D=32 cm) and F_{max}=66.18 kN (D=192 cm), respectively. The normalized nominal (flexural) strength $\sigma_n/f_t=1.5F_{max}L/(D^2tf_t)$ varied between 1.1 (D=192 cm) and 1.5 (D=8 cm). The strength and ductility strongly increased with decreasing beam height. For the large and very large-size beam, the snap-back behaviour occurred (decrease of strength with decreasing deformation). It was in particular very strong for the very large-size beam.

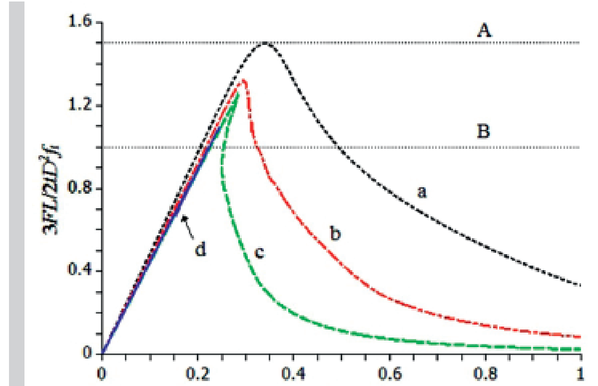

Figure 4.
Normalized horizontal normal (flexural) stress - deflection curves $1.5FL/(f_tD^2t)=f(u/D)$ under 3-point bending with constant values of tensile strength for 4 different concrete beam heights: small D=8 cm (dashed line "a"), medium D=16 cm (dotted-dashed line "b"), large D=32 cm (dotted line "c"), very large D=192 cm (solid line "d"), A) line σ_N/f_t=1.5, B) line σ_N/f_t=1.0 (F – vertical force, L – beam length, D – beam height, t – beam thickness, f_t – tensile strength)

The width of a localized zone for all beam sizes was about w=1.5 cm. In turn, the height of the localized zone h measured at the peak load increased non-linearly with increasing beam height D, i.e.: 24 mm, 34 mm, 40 mm, and 48 mm for the small (D = 80 mm), medium (D = 160 mm), large (D = 320 mm) and very large beam (D=1920 mm), respectively. The larger the beam, the lower was the ratio of the localized zone height to the beam height h/D: 0.3 (D=80 mm), 0.212 (D=160 mm), 0.125 (D=320 mm) and 0.025 (D=1920 mm).

In Fig. 5, the evolution of the normalized flexural stress $1.5FL/(f_tD^2t)$ as the function of the scale parameter h/D is presented. When the parameter h/D increases, both the nominal strength and brittleness of concrete beams also increase.

Figure 6 shows the outcomes from deterministic simulations indicating a pronounced deterministic-energetic size effect on the normalized flexural strength $f_r/f_t=1.5F_{max}L/(D^2tf_t)$. The nominal strength of

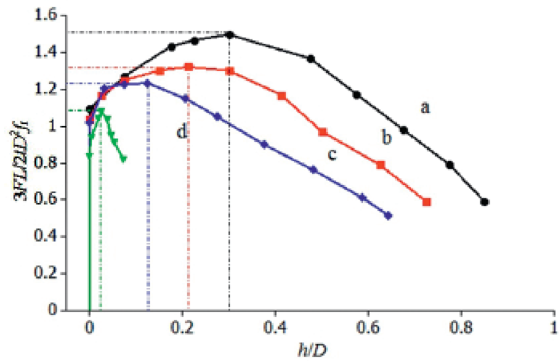

Figure 5.
Evolution of normalized horizontal normal (flexural) stress $1.5FL/(f_tD^2t)$ versus normalized localized zone height h/D in deterministic simulations for concrete beams ("a" – small, "b" – medium, "c" – large and "d" – very large-size beam)

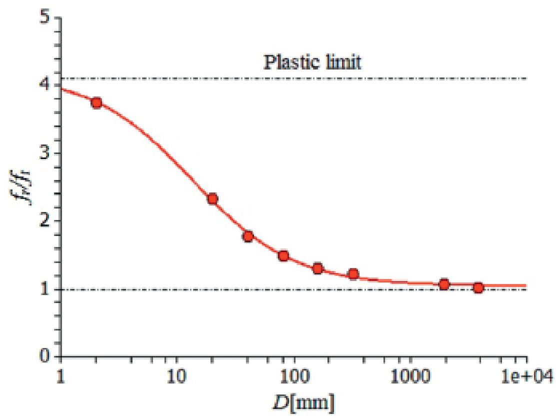

Figure 6.
Calculated normalized flexural tensile strength f_r/f_t=$1.5F_{max}L/(f_tD^2t)$ versus beam height D in unnotched concrete beams from deterministic FE calculations (red circles) versus beam height D compared with the deterministic size effect model by Bazant (red solid line by Eq. 1)

unnotched concrete beams approaches the horizontal asymptote for very large structures. On the basis of the nonlinear regression method by Leveneberg-Marquardt, the following parameters were found to fit Eq.1: f_r^∞=3.782 MPa, D_b=40 mm, l_p=13.6 mm, r=1.0. The agreement of FE results for 8 beams with Eq. 1 is almost perfect.

5.2. Statistical size effect

The 12 different evolutions of the normalized vertical force $1.5FL/(f_tD^2t)$ versus the normalized vertical deflection u/D are shown in Fig. 7 (the deterministic curve is also attached).

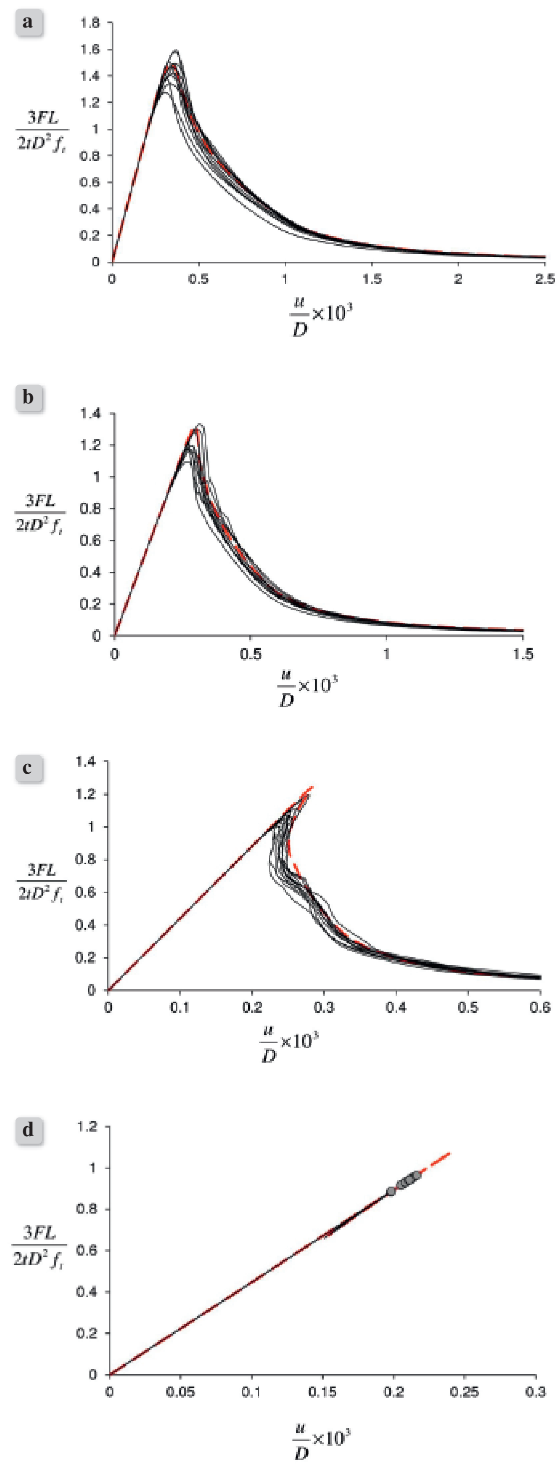

Figure 7.
Normalized horizontal normal (flexural) stress-deflection curves $1.5FL/(f_tD^2t)$=f(u/D) with constant (dashed red line) and random (solid lines) value of tensile strength for 4 different beam heights: a) small-size D=8 cm, b) medium-size D=16 cm, c) large-size D=32 cm, d) very large-size beam D=192 cm

The deterministic normalized vertical force is located in the range of stochastic values for a small and medium-size beam or is the maximum values for a large and very large-size beam. For the height of $D=8$ cm, the maximum vertical force changes between $F_{max}=3.267\text{-}4.08$ kN, and the mean value $F_{mean}=3.72$ kN is by 3% smaller than the deterministic value $F=3.83$ kN (the coefficient of variation $cov=0.063$). If the beam height is $D=16$ cm, the maximum vertical force varies between 5.61-6.82 kN and the mean stochastic force $F_{mean}=6.25$ kN (with the coefficient of variation $cov=0.057$) is smaller by 7% than the deterministic value ($F=6.75$ kN). For the both beams, the single maximum statistical vertical force can be higher than the deterministic one. If the beam height is $D=32$ cm, the maximum vertical force changes between 10.31-12.25 kN, and the mean stochastic $F_{mean}=11.07$ kN (with the coefficient of variation $cov=0.053$) is smaller by 12% than the deterministic value of $F=12.57$ kN. Finally, in the case of the very large-size beam $D=192$ cm, the maximum vertical force changes between 54.32-59.18 kN and the mean stochastic $F_{mean}=57.14$ kN (the variation coefficient equals $cov=0.027$) is smaller by 14% than the deterministic value of $F=66.18$ kN. Thus, both the mean stochastic nominal strength and coefficient of variation always decrease with increasing size D and the influence of the random distribution of the tensile strength on the nominal strength is stronger for larger structures.

The random distribution of f_t does not affect the mean width of a localized zone, which is again about 1.5 cm for all beam sizes. A localized zone can be strongly non-symmetric and curved. It occurs at the mean distance of about 2.0 cm (small-size beam) and of about 40 cm (very large-size beam) from the beam-centre. The mean height of localized zones h at peak was closed to the deterministic outcomes.

The maximum vertical force in concrete beams strongly depends on the position of a localized zone. This position is connected with the distribution and magnitude of the tensile strength at the place of a localized zone (within the area $w\times h$) and the magnitude of the horizontal normal stress σ_{11} due to a bending moment. A localized zone is created, where the ratio of mean local tensile strength $\bar{f}_t$ in the localized area $w\times h$, to normal stress σ_{11} is minimum. The maximum vertical force increases with increasing ratio $\bar{f}_t(w\times h)/\sigma_{11}$. In a small-size beam, the beam mid-region where a localized zone can be created is very limited due to the assumed standard deviation of the tensile strength and correlation range (3 cm in a vertical direction and 8 cm in a horizontal direction). In this limited beam region (with a small number of weak spots), the tensile strengths are strongly correlated and can be higher or lower than its mean value $f_t=3.6$ MPa. Therefore, the vertical normal tensile force can be smaller or larger than this in the deterministic study (depending on the spot choice by a localized zone for propagation). With an increase of the beam size, the number of weaker local spots increases with the correlation range assumed and the beam mid-region where a localized zone can propagate is significantly larger. In this wide beam region, the tensile strengths are weaker correlated than in a small-size beam. So there exists a very high probability to achieve a smaller vertical force than in a small beam due to the great number of weak spots with the tensile strength smaller than $f_t=3.6$ MPa, which can be chosen by a localized zone for propagation.

Bazant and Novak [21], [36] recommended the Weibull modulus $m=24$ in Eq. 5 (with $L_o=D_b=15.53$ mm, $l_p=0$, $r=1.14$ and $f_r^{\infty}=3.68$ MPa). Our deterministic-statistical results show the best agreement with Eq. 5 by assuming the Weibull modulus $m=48$ based on the coefficient of variation $cov=0.027$ from Eq. 6 (with $D=192$ cm) and $L_o=D_b=30.37$ mm, $n=2$, $l_p=0$, $r=1$ and $f_r^{\infty}=3.90$ MPa.

Figure 8 presents a comparison between our numerical deterministic-statistical results and size effect law by Bazant (Eqs. 5) with the related asymptotes using the Weibull modulus $m=12\text{-}48$. The deterministic-statistical outcomes indicate a further decrease of the nominal (flexural) strength with increasing beam size while the deterministic ones reach their lower limit. Our deterministic-statistical results present also a satisfactory agreement with the size effect law by Bazant (Eq. 5) by assuming the recommended value of $m=24$ ($L_o=D_b=16.95$ mm, $l_p=0$, r=1 and $f_r^{\infty}=4.753$ MPa).

Next, we used the deterministic parameters from Section 5.1 ($D_b=40$ mm, $l_p=13.6$ mm, $r=1$ and $f_r^{\infty}=3.78$ MPa, $n=2$) which fit Eq. 1 to match now Eq.5. Only the Weibull modulus $m=48$ enabled a transition from a pure deterministic to a coupled deterministic-statistical size effect. The value $m=12$ highly underestimated the calculated deterministic-statistical flexural tensile strength.

All size effect results of the normalized nominal (flexural) strength $\sigma_N/f_t=1.5F_{max}L/(D^2tf_t)$ for

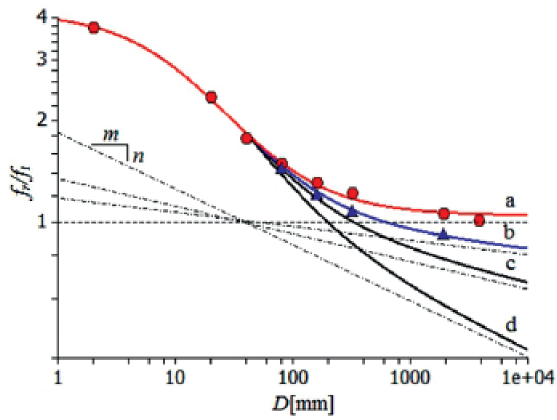

Figure 8.
Calculated normalized flexural tensile strength $f_r/f_t=1.5F_{max}L/(f_tD^2t)$ versus beam height D from deterministic (circles) and stochastic (triangles) FE calculations compared with deterministic (line "a", Eq. 13) and deterministic-stochastic size effect law by Bazant (Eq. 17) for various Weibull moduli m and constant deterministic parameters (line "b" – m=48, line "c" – m=24, line "d" – m=12)

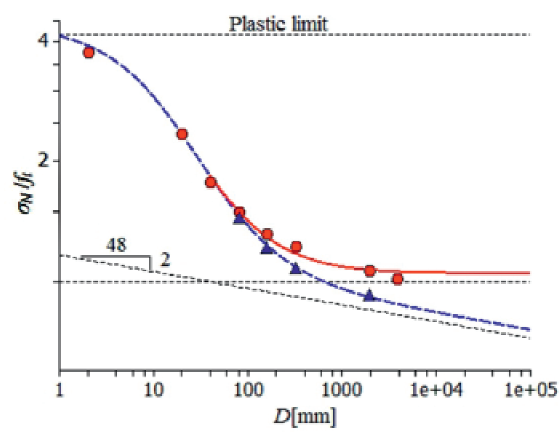

Figure 9.
Calculated normalized nominal (flexural) strength $\sigma_N/f_t(\sigma_N=1.5F_{max}L/(f_tD^2t))$ versus beam height D for unnotched concrete beams from deterministic (red circles) and stochastic (blue triangles) FE calculations compared with deterministic (Eq. 1) (red solid line) and deterministic-stochastic (Eq. 5) (blue dashed line) size effect law by Bazant

unnotched concrete beams are summarized in Fig. 9 as compared to the size effect laws by Bazant (Eq. 5) with m=48, L_o=D_b=40 mm, l_p=13.6 mm, r=1, f_r^∞=3.78 MPa and n=2.

6. CONCLUSIONS

The following conclusions can be drawn from our non-linear FE-analysis with the constant and spatially correlated local stochastic tensile strength for unnotched plain concrete beams of a similar geometry.

Our numerical approach is capable to describe a deterministic and statistical size effect. The calculated combined deterministic-statistical size effect is in agreement with the size effect law SEL and MFSL in the considered beam size range. However, fractality is not needed to induce a size effect, since the stress redistribution and energy release during strain localization cause a size effect (thus, fractality can contribute to a certain refinement of a size effect but not to its replacement). The size effect model by Bazant is more universal and has physical foundations, and can be introduced into design codes.

A decrease of the nominal strength with increasing beam size is significant in deterministic calculations. The flexural tensile strength measured at laboratory scale is highly overestimated. The snap-back behaviour occurs already in large-size beams with the height of 32 cm. The width of the localized zone is about 1.5 cm for all beam sizes. The ratio of the height of the localized zone to the beam height decreases with increasing beam size. The localized zone is straight.

A further decline of the nominal strength with increasing beam size is caused by a random distribution of the tensile strength. The larger the beam, the stronger is the influence of a statistical distribution on the nominal strength due to the presence of a larger number of local weak spots (i.e. the mean statistical bearing capacity is always smaller than the deterministic one). The statistical bearing capacity is larger in some realizations with small and medium-large beams than the deterministic value. The randomness of the tensile strength does not change the mean width of the localized zone. The localized zone can be curved and non-symmetric. This position of the localized zone is connected with the distribution and magnitude of the tensile strength in a localized zone at peak and the quantity of the horizontal normal stress due to bending. In corresponding notched concrete beams, the stochastic size effect is negligible.

Our FE results match well the combined deterministic-statistical size effect law by Bazant with the Weibull modulus m=24-48. In turn, a prediction of the combined deterministic-statistical size effect based on deterministic results is only possible with the modulus m=48.

ACKNOWLEDGMENT

Scientific work has been carried out by the first two authors as a part of the Project: "Innovative resources and effective methods of safety improvement and durability of buildings and transport infrastructure in the sustainable development" financed by the European Union (POIG.01.01.02-10-106/09-01)

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