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THE INFLUENCE OF THE INJECTION SOURCE ON THE FOUNDATION IN CONDITIONS OF THE ORTHOTROPIC FLOW OF FLUID

Jan GASZYŃSKI [a]*, Karolina ŁACH [b]

[a] Associate Prof.; Institut of Geotechnics, Cracow University of Technology
E-mail address: *jgaszyn@usk.pk.edu.pl*

[b] Dr.; Institut of Geotechnics, Cracow University of Technology

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Abstract
The subject of the paper work is reflection on the foundation and ground interaction in case of action of the pressure source. The soil has different characteristic of flow in the horizontal and vertical directions. The result is the relationship between the movements of the foundation and effect of the pressure source located in the soil. It is assumed: two-phase model. It is built to a homogenous, linear-elastic skeleton and fluid, which fills completely the ground. The fluid flow is laminar, it applies to the Darcy flows law. The skeleton and the fluid are mutually coupled. The solutions are the foundation displacement and stress in the contact zone.

Streszczenie
Przedmiotem pracy są analiza współdziałania fundamentu i podłoża gruntowego w przypadku działania w podłożu źródła ciśnienia (np. iniekcji).
Do rozważania zadania przyjęto model konsolidacji dwufazowego ośrodka gruntowego, w którym zachowanie szkieletu opisuje model liniowo sprężysty, a ruch cieczy, całkowicie wypełniającej pory gruntowe ma charakter laminarny. Przyjęto zróżnicowane właściwości przepływu w kierunkach: poziomym i pionowym. Rezultatem rozwiązanego zadania brzegowego są zależności pomiędzy przemieszczeniem fundamentu i działaniem źródła ciśnienia usytuowanego w podłożu. Ponadto podano naprężenia w kontaktowe w funkcji ciśnienia w źródle.

Keywords: **Consolidation; Source of pressure; Orthotropic flow.**

1. INTRODUCTION

A subject of paper is a connection between the subsidence of the foundation and the power of the source in different conditions of the flow (in a variety of flow conditions).

To describe the deformation and stress distribution model of consolidation was used. This model takes into account the source of disturbance and the orthotropic properties of the flow, resulting from different ground permeability in the vertical and horizontal directions.

In considered issues assumptions of the two-phase soil medium were made. It is built of homogeneous, linear-elastic skeleton and fluid that fills completely the pores of ground.

The motion of the pore fluid is laminar, apply to the Darcy flows law. Deformation of the skeleton and the pressure of the pore fluid are mutually interrelated.

Problems were solved in the axially-symmetric state of the deformation.

2. EQUATIONS SYSTEM

Designations adopted in the work:

u, w – coordinate of displacement vector in the direction of r (radius) and the axis z

ε_r, ε_z, ε_φ, ε_{rz} – coordinates of the skeleton deformation tensor

θ – dilatation of the fluid

N,Q,R,M – parameters of the porous medium

k_r, k_z – permeability coefficients in direction of the radius r (horizontal) and of axis z (vertical)

$\eta^2 = \frac{k_r}{k_z}$ – degree of the anisotropy [1]

V^o – action of the source in the initial moment

$V(t)=V^oH(t)$ – function describing the effect (operation) of the source

$P(t)=P\cdot H(t)$ – force causes settlement of the foundation

σ_r, σ_z, σ_{rz} – coordinates of the stress tensor in the skeleton

σ – pressure of the fluid in the pores

Parameters and relationships between parameters for the porous body [2]:

$$A = M + \frac{Q^2}{R}$$

$$H = Q + R$$

$$E_k = M + 2N$$

$$E_p = \frac{R(M+2N)+H^2}{R} = E_k + \frac{H^2}{R}$$

$$a = \frac{R(M+N)+H^2}{R(M+2N)+H^2}$$

$$B = \frac{R^2(M+2N)}{R(M+2N)+H^2} = R\frac{E_k}{E_p}$$

$$B_o = \frac{R(M+2N)}{H} = \frac{R}{H}E_k$$

$$p = \frac{1}{k_z B} \qquad \bar{S}(s) = s\bar{V}(s) - V^o$$

$$\mu^2 = \eta^2\omega^2 + \frac{s}{k_z B} = \eta^2\omega^2 + ps$$

$$\overline{M}(s) = a - \frac{NH}{B_o E_p}\left(1 - \frac{ps}{(\omega+\mu)^2}\right)$$

$$b = 1 - \frac{HN}{B_o E_p a}$$

Condition of consolidation body, in case of operation of source of pressure, describes the following system of equations [3]. It was saved in a cylindrical coordinate system (0,r,φ,z), in axially symmetric deformation state:

equations of displacement of skeleton:

$$N\left(\Delta - \frac{1}{r^2}\right)u + (N+M)\varepsilon,_r + \frac{H}{R}\sigma,_r = 0 \qquad (1)$$

$$N\Delta w + (M+N)\varepsilon,_z + \frac{H}{R}\sigma,_z = 0 \qquad (2)$$

fluid flow equation:

$$k_r\sigma,_{rr} + k_r\frac{1}{r}\sigma,_r + k_z\sigma,_{zz} = \frac{1}{R}\dot{\sigma} - \frac{H}{R}\dot{\varepsilon} + V \qquad (3)$$

$$V = \frac{\delta(r)}{r}\cdot\delta(z)\cdot\tau(t)\cdot V^o \qquad (4)$$

geometrical relationships:

$$\varepsilon_r = \frac{\partial u}{\partial r} \qquad \varepsilon_\Phi = \frac{u}{r} \qquad \varepsilon_z = \frac{\partial w}{\partial z} \qquad (5)$$

$$\varepsilon = \frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} \qquad \varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right)$$

Constitutive equations:

$$\sigma = Q\cdot\varepsilon + R\cdot\theta \qquad (6)$$

$$\sigma_{ij} + \sigma\delta_{ij} = 2N\varepsilon_{ij} + \left(M\varepsilon + \frac{H}{R}\sigma\right)\delta_{ij} \qquad (7)$$

boundary conditions:

$$w = w_o \qquad r < r_o \qquad (8)$$

$$\sigma_z + \sigma = 0 \qquad r > r_o \qquad (9)$$

$$\sigma_{rz} = 0 \qquad r > 0 \qquad (10)$$

$$\sigma = 0 \qquad r > 0 \qquad (11)$$

Initial conditions for $t=t_o$:

– compatibility equations:

$$N\left(\Delta - \frac{1}{r^2}\right)u^o + (M+N)\varepsilon,_r^o + \frac{H}{R}\sigma,_r^o = 0 \quad (12)$$

$$N\Delta w^o + (M+N)\varepsilon,_z^o + \frac{H}{R}\sigma,_z^o = 0 \quad (13)$$

– The equation describing the motion of the fluid filling the pores of skeleton:

$$\left(k_r \frac{\partial^2}{\partial r^2} + k_r \frac{1}{r}\frac{\partial}{\partial r} + k_z \frac{\partial^2}{\partial z^2}\right)\sigma^o = V^o \frac{\delta(r)}{r}\delta(z) \quad (14)$$

A problem was solved applying integral transformations and the operator calculation. Details of solutions can be found at work [4].

The solution for the foundation, which cooperates with the consolidation body was searched for. Coordinates of the stress state in the contact area are not known.

The function of contact stresses $q(r,\ t)$ has to satisfy the boundary conditions (8), (9) and the expression:

$$\int_0^{2\pi}\int_0^{r_o} (\sigma_z + \sigma) dr d\varphi = -P(t) \quad (15)$$

Where P is a total load transmitted by the foundation to the soil.

Conditions (8) and (9) were converted and solutions were received:

$$\int_0^\infty \frac{\overline{q}'(\omega,s) - \overline{\tau}'(\omega,s)}{\overline{M}(s)} J_o(\omega r) d\omega = 2N\overline{w}_o \qquad r < r_o \quad (16)$$

$$\int_0^\infty \overline{q}'(\omega,s)\omega J_o(\omega r) d\omega = 0 \qquad r > r_o \quad (17)$$

Solving above equations meets a difficulty at inverting integral transformations. Therefore, complicated formulations have been replaced by analytical approximations that retain asymptotic properties of solutions.

$$\overline{\tau}'(\omega,s) = \frac{4N}{B_o k_z s}\left[V^o + \frac{\overline{S}(s)}{1+psh^2}\right]\frac{\exp(-\omega h) - \exp(-\eta\omega h)}{\omega(\eta^2-1)} \quad (18)$$

$$\frac{1}{\overline{M}(s)} = \frac{\eta + psh^2}{a(\eta + psh^2) - \eta\dfrac{HN}{B_o E_p}} \quad (19)$$

It is searched for the function $\overline{q}'(\omega,s)$ in the form:

$$\overline{q}'(\omega,s) = \int_0^{r_o} \overline{g}(\xi,s)\cos(\omega\xi) d\xi \quad (20)$$

It identity fulfill the equation (17).

After inserting to the equation (16) it is possible to receive:

$$\int_0^r \frac{\overline{g}(\xi,s)}{\sqrt{r^2-\xi^2}} d\xi - \frac{4N}{B_o k_z s}\left[V^o + \frac{\overline{S}(s)}{1+psh^2}\right]\int_0^\infty \frac{\exp(-\omega h) - \exp(-\eta\omega h)}{\omega(\eta^2-1)} J_o(\omega r) d\omega = 2N\overline{M}(s)\overline{w}_o \quad (21)$$

After calculations:

$$\overline{g}(\xi,s) = \frac{4N}{\pi}\overline{M}(s)\overline{w}_o + \frac{2}{\pi}\frac{4N}{B_o k_z s}\left[V^o + \frac{\overline{S}(s)}{1+psh^2}\right]\frac{1}{\eta^2-1}\frac{1}{2}\ln\frac{\xi^2+\eta^2h^2}{\xi^2+h^2} \quad (22)$$

The expression (22) enables to appoint the contact stress, the line of the deformation of the boundary and settlement of the foundation in function of loading and action of the source.

Expressions for contact stresses and the displacement of the boundary are as follows:

$$\begin{aligned}\overline{q}(r,s) = {} & \frac{4N}{\pi}\left[\overline{M}(s)\overline{w}_o + \frac{1}{B_o k_z s}\left[V^o + \frac{\overline{S}(s)}{1+psh^2}\right]\frac{1}{\eta^2-1}\ln\frac{r_o^2+\eta^2h^2}{r_o^2+h^2}\right]\frac{1}{\sqrt{r_o^2-r^2}} + \\ & + \frac{4N}{\pi B_o k_z s}\left[V^o + \frac{\overline{S}(s)}{1+psh^2}\right]\frac{1}{\eta^2-1}\left[\frac{1}{\sqrt{h^2+r^2}} arctg\frac{\sqrt{r_o^2-r^2}}{\sqrt{h^2+r^2}} + \right. \\ & \left. - \frac{1}{\sqrt{\eta^2h^2+r^2}} arctg\frac{\sqrt{r_o^2-r^2}}{\sqrt{\eta^2h^2+r^2}}\right]\end{aligned} \quad (23)$$

$$\begin{aligned}\overline{w}(r,s) = {} & \frac{2}{\pi}\overline{w}_o \arcsin\left(\frac{2r_o}{r_o+r+|r_o-r|}\right) + \\ & + \frac{2}{\pi B_o k_z s\overline{M}(s)}\left[V^o + \frac{\overline{S}(s)}{1+psh^2}\right]\frac{1}{\eta^2-1}\int_0^r \ln\frac{\xi^2+n^2h^2}{\xi^2+h^2}\frac{1}{\sqrt{r^2-\xi^2}} d\xi + \\ & + \frac{2}{\overline{M}(s)B_o k_z s}\left[V^o + \frac{\overline{S}(s)}{1+psh^2}\right]\frac{1}{\eta^2-1}\ln\left|\frac{\sqrt{h^2+r^2}+h}{\sqrt{\eta^2h^2+r^2}+\eta h}\right|\end{aligned} \quad (24)$$

Let $P(t)$ mean loading of the foundation to the soil, and $\overline{P}(s)$ means the Laplace transformation of this loading, then:

$$P(t) = \int_0^{2\pi}\int_0^{r_o} q(r,t) r d\varphi dr \quad (25)$$

Substituting (25) to (23) and calculating the integral it is possible to receive:

$$\overline{P}(s) = 8N\overline{M}(s)\overline{w}_o r_o + \frac{16NV^o}{B_o k_z s}\frac{h}{\eta^2-1}\left[\frac{r_o}{2h}\ln\frac{r_o^2+\eta^2h^2}{r_o^2+h^2} + \eta ar\,\mathrm{ctg}\frac{r_o}{\eta h} - ar\,\mathrm{ctg}\frac{r_o}{h}\right] \quad (26)$$

And:

$$\overline{w}_o = -\frac{2V^o}{B_o k_z s\overline{M}(s) r_o}\frac{h}{\eta^2-1}\left[\frac{r_o}{2h}\ln\frac{r_o^2+\eta^2h^2}{r_o^2+h^2} + \eta ar\,\mathrm{ctg}\frac{r_o}{\eta h} - ar\,\mathrm{ctg}\frac{r_o}{h}\right] + \frac{\overline{P}(s)}{8N\overline{M}(s) r_o} \quad (27)$$

After inserting Laplace transforms and taking into account (19):

$$w_o = \frac{-V^o}{B_o k_z ab}\frac{2h}{r_o(\eta^2-1)}\left[\frac{r_o}{2h}\ln\frac{r_o^2+\eta^2h^2}{r_o^2+h^2}+\eta\, ar\,ctg\frac{r_o}{\eta h}-ar\,ctg\frac{r_o}{h}\right]\cdot$$
$$\cdot\left[1-(1-b)\exp\left(-\frac{\eta bt}{ph^2}\right)\right]H(t)+\frac{P}{8Nar_o b}\left[1-(1-b)\exp\left(-\frac{\eta bt}{ph^2}\right)\right]H(t) \tag{28}$$

Let $\overline{w}_o=0$ in the expression (23) and it is possible to receive the transform of contact stresses in the plain z=0, which after inserting the Laplace transformation is:

$$q(r)=\frac{4NV^o}{\pi B_o k_z}\frac{1}{\eta^2-1}\left[\frac{1}{\sqrt{h^2+r^2}}ar\,ctg\frac{\sqrt{r_o^2-r^2}}{\sqrt{h^2+r^2}}-\frac{1}{\sqrt{\eta^2h^2+r^2}}ar\,ctg\frac{\sqrt{r_o^2-r^2}}{\sqrt{\eta^2h^2+r^2}}+\right.$$
$$\left.+\frac{1}{\sqrt{r_o^2-r^2}}\ln\frac{r_o^2+\eta^2h^2}{r_o^2+h^2}\right] \tag{29}$$

All calculations were performed for the following parameter values (assumed physical parameters for the silty clay in a plastic state) [5]:

N=0.5 MPa, A=1.1 MPa, Q=1.1 MPa, R=4.0 MPa,

$k=8\cdot10^{-3}\dfrac{m^4}{kN\cdot 24h}$, p_o= 0.3 MPa – pressure in the source

3. THE CONSOLIDATING HALF-SPACE OF THE SOIL WITH THE FOUNDATION

It is assumed that there is situated a rigid foundation of the circled base and the radius r_o on the boundary of the consolidating half-space.

The force P causes settlement of the foundation:

$$w_p(t)=\frac{P}{8Nar_o b}\left[1-(1-b)\exp\left(-\frac{\eta bt}{ph^2}\right)\right]H(t) \tag{30}$$

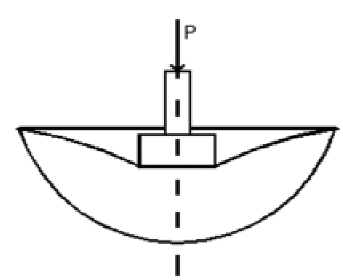

Figure 1.
Cross section of the foundation and the soil

In Fig. 2 there are settlements of the foundation in the function of time and the distance from the centre of the foundation.

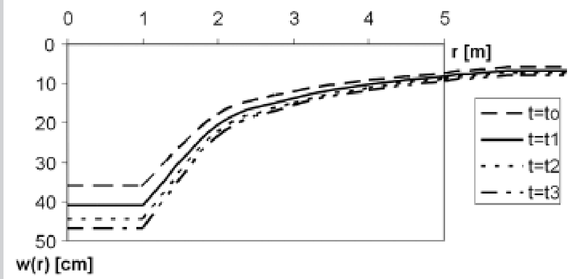

Figure 2.
Displacement of the foundation in the function of r and time

Using the expression (30) it is possible to check as the orthotropic flow of fluid influences the settlement in function of time. Fig. 3 shows changes in the settlement depending on the coefficient of anisotropy.

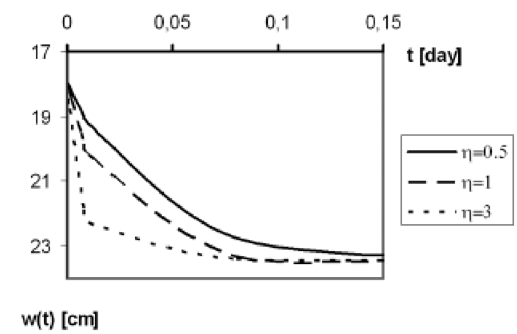

Figure 3.
Displacement in the function of coefficient of anisotropy

4. HALF-SPACE UNDER THE EFFECT OF THE LOADING AND THE SOURCE OF THE PRESSURE

Applying the injection (represented here by the source of the pressure), it is possible to prevent too big settlement. To increase the load capacity and reduce the strain of the soil (Fig. 4), together with the building of the foundation initiates a pressure source located at the point (0, h), described by the expression:

$$V(t)=\frac{k_z p_o}{h}H(t) \tag{31}$$

Action of the source is changing stresses in the contact area of the foundation and soil as well as is changing the nature of settlement.

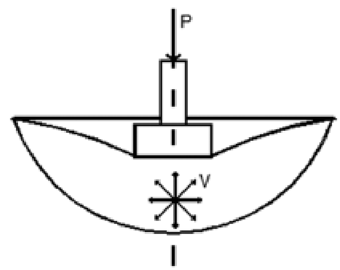

Figure 4.
Cross section of the foundation and the soil with the source of pressure

Expressions for the contact stress and settlement are described by (28) and (29).

Fig. 5 shows the displacement of the foundation under load and the operation of the source as a function of time and distance from the base of the foundation (r_o=1m, h=1m).

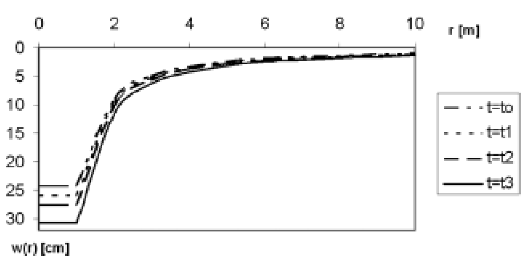

Figure 5.
Displacement as a function of time and distance of foundation

Influence the orthotropic flow for settlement of the foundation was described on Fig. 6

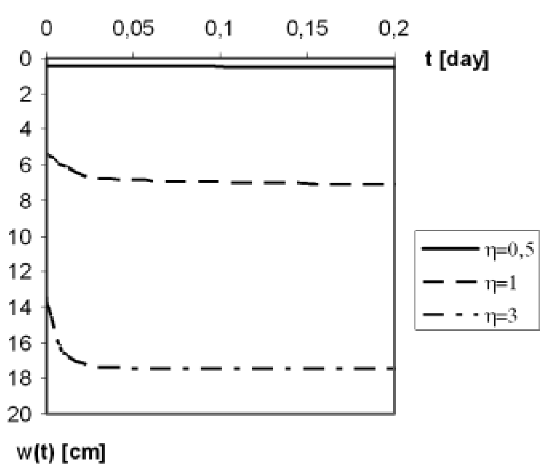

Figure 6.
Displacement in function of coefficient of anisotropy

5. HALF-SPACE UNDER THE EFFECT OF THE LOAD AND THE SOURCE OF THE PRESSURE REDUCING SETTLEMENT

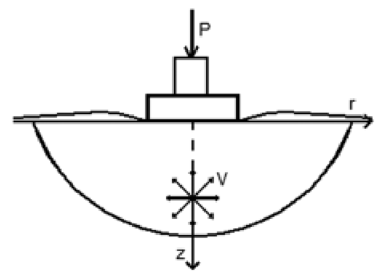

Figure 7.
The foundation situated in the plain z=0

The initiation of the sources of the pressure can change both the course of settlement as well as completely eliminate settlement (Fig. 7). For that purpose an interaction of the foundation and the source is being considered. The loading of the foundation is being balanced by action of the source, on the depth h. Settlement of the foundation determined by the expression (27) is equaling zero. For so formulated conditions the value of the final operation of the source is:

$$V=\frac{B_o k_z}{16N}\frac{\eta^2-1}{h}\frac{P}{\frac{r_o}{2h}\ln\frac{r_o^2+\eta^2h^2}{r_o^2+h^2}+\eta arctg\frac{r_o}{\eta h}-arctg\frac{r_o}{h}} \tag{32}$$

Also for these conditions, using equation (31), the pressure in the source is:

$$p_o=\frac{B_o}{16N}\left(\eta^2-1\right)\frac{P}{\frac{r_o}{2h}\ln\frac{r_o^2+\eta^2h^2}{r_o^2+h^2}+\eta arctg\frac{r_o}{\eta h}-arctg\frac{r_o}{h}} \tag{33}$$

Fig. 8 shows the relationship between the foundation and the power of the source for various coefficients η.

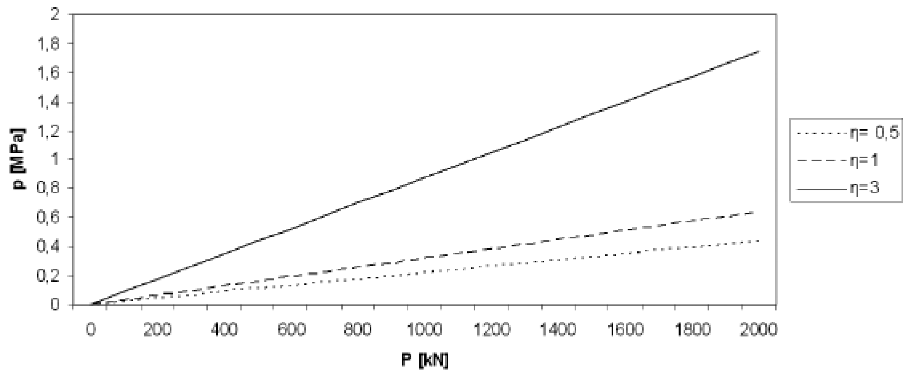

Figure 8.
Power of the source in function of load of foundation and coefficient of anisotropy

It is possible to check how the depth of situating the injection affects the pressure in the source (Fig. 9), as well as how the pressure in the source is changing for different flow conditions (Fig. 10).

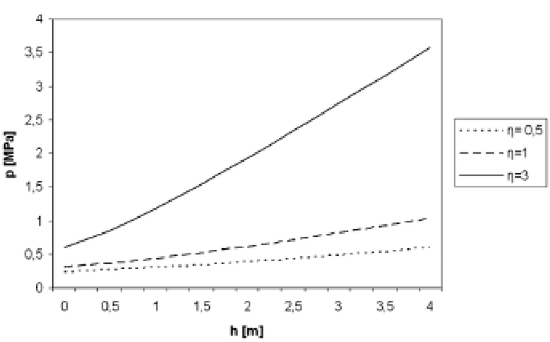

Figure 9.
Power of the source in function of its depth and coefficient of anisotropy

6. CONCLUSIONS

At the paper work the influence of the orthotropic flow on the cooperation of the foundation and soil was being analysed. Obtained results allow to control the course of settlement in function of the time for different flows (28). The expression (32) allows to predict the power of the source for different soil.

Using the expression (33) it is possible to evaluate injection in order to soil reinforcement. It is possible to select the final pressure in the source which is supposed to reduce settlement of the foundation in function of geotechnical parameters, the depth on which the source is situated and the pressure of the foundation on soil.

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