

## MODELING OF DISCONTINUOUS BOND UNDER THE EQUAL LOAD IN RC BEAM STRENGTHENED WITH CFRP PLATE

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### Abstract

Decrease of the bearing capacity of constructions imposes the need of their repair and strengthening. In the recent years, typical retrofitting technique involves use of external bonded FRP (fiber reinforced polymers) reinforcement. One of the problems that can be encountered during the strengthening in the practice is inadequate execution of the bonding process. This may lead to weakening of the bond layer and to creating discontinuities zones between reinforced concrete and CFRP plate. The bond between reinforced concrete beam and CFRP plate in this paper is modelled with the use of a numerical displacement-based fiber model, which is based on the concept of different displacement fields for concrete and reinforcing plate and therefore takes into account bond-slip effects, which is essential for the realistic prediction of the beam behavior. Discontinuous bond zone is modelled by modification in the constitutive law for description of the bond between the reinforced concrete beam and CFRP plate. In this paper, verification of implemented modification in the constitutive law is presented for two different cases of discontinuous zones in the bond layer. The weak bond behaviour for two cases is analysed under the equal load dependent on different increasing displacement at slip while the maximum shear stress remains unchanged. Implemented modification is analysed using bond stress distribution and tensile plate force distribution along the externally strengthened reinforced concrete beam. Numerical analysis carried out shows that the proposed modeling of the weak zone is appropriate.

### Streszczenie

Redukcja nośności konstrukcji powoduje potrzebę ich naprawy i wzmocnienia. Ostatnimi czasy typowe techniki naprawy bazują na wykorzystaniu zewnętrznie przyspójonego zbrojenia FRP (polimery wzmocniane włóknami). Jednym z problemów jaki można napotkać w praktyce podczas wzmocniania jest niewłaściwie wykonany proces spajania. Może to prowadzić do osłabienia warstwy łączącej i tworzenia się nieciągłych stref pomiędzy powierzchnią elementu żelbetowego i taśmą CFRP. W artykule połączenie pomiędzy żelbetową belką a taśmą CFRP zamodelowano przy użyciu numerycznego modelu bazującego na przemieszczeniach, którego idea opiera się na założeniu różnych pól przemieszczeń dla betonu i taśmy wzmocniającej. Pozwala to uwzględnić efekt poślizgu połączenia, który jest istotny dla przewidywania rzeczywistego zachowania się belki. Strefa nieciągłości połączenia jest modelowana poprzez modyfikację związków konstytutywnych opisujących połączenie pomiędzy żelbetową belką i taśmą CFRP. W artykule przedstawiono weryfikację zastosowanej modyfikacji związków konstytutywnych na dwóch różnych przykładach stref nieciągłości w warstwie łączącej. W obu przypadkach zachowanie się osłabionego połączenia analizowano pod jednakowym obciążeniem zależnym od narastającego przemieszczenia przy poślizgu, podczas gdy maksymalne wartości naprężenia ścinającego pozostawały niezmiennic. Zastosowaną modyfikację analizowano przy użyciu rozkładu naprężeń w połączeniu oraz rozkładu sił rozciągających w płaszczyźnie zewnętrznego wzmocnienia żelbetowej belki. Przeprowadzona analiza numeryczna pokazuje, że zaproponowany sposób modelowania osłabionej strefy jest właściwy.

Keywords: CFRP; Numerical analysis; Reinforced concrete beam; Strengthening; Weak zone.

## 1. INTRODUCTION

Performance of the materials used in the contemporary structures can significantly change as a result of change in the environmental conditions and the increase of the loads, which were not taken into account in the design process. All these factors may decrease the bearing capacity or structural safety of the construction during its service life. As a consequence, non adequate performance of the constructions imposes the need for their repair and strengthening. Increasing the load capacity and structural safety of the flexural loaded structural members is often carried out by external bonding of additional reinforcement. In the recent years, typical retrofitting technique involves the use of externally bonded lighter, stronger and more durable FRP (fibre reinforced polymer) strips.

In order to achieve successful external strengthening of the reinforced concrete structures by FRP strips a thorough understanding of the effects that this type of FRP reinforcement has on beam failure mode is required. Key role in the failure of the externally strengthened construction plays the bond layer between reinforced concrete surface and FRP strip. Experimental researches show that the most often type of failure of the strengthened construction, caused by the maximal shear stresses, is followed by peeling of the FRP strip initiated at the end of the plate, where concrete is uncracked. Local shear failure is driven by a biaxial tension state composed by the interfacial stresses and the normal tension induced on concrete by the flexure [1]. From the theory proposed by Taljsten [2], it can be concluded that for the cases of sufficiently thin strengthening plates, the influence of the peeling stresses on the principal stresses is minute and thus can be neglected.

One of the problems that can be encountered during the strengthening of reinforced concrete structures in the practice is inadequate execution of the bonding process. This may lead to weakening of the bond layer in some positions along the length of the plate, and to creating discontinuities within the bond layer.

In order to determine properly with adequate accuracy the bearing capacity of the reinforced concrete structure strengthened with externally added FRP reinforcement a model need to be used, which can properly describe the stresses in the bond layer [3]. In the recent years in the field of modelling flexural loaded strengthened reinforced concrete elements great progress has been achieved [4]. In the field of research and education very often beam models

based on the concept of discretization of the cross section into fibbers layers are used. Fibber models in the same time take into account axial and flexural influence. These models could be combined with any model of beam element based on displacement method, as well as based on the force method. With appropriate modifications fibber model could be used for the analysis of the reinforced concrete elements strengthened with externally added FRP plates.

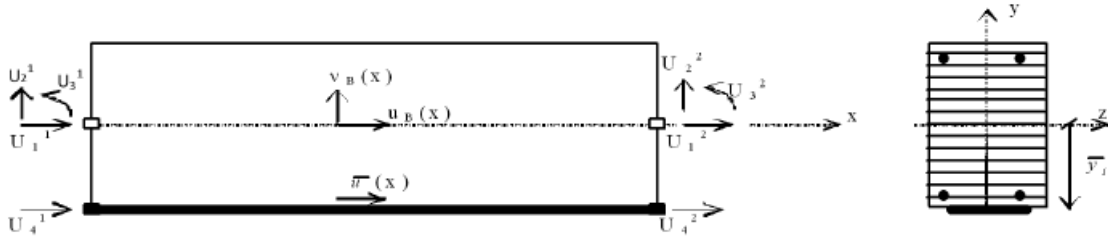
The bond between reinforced concrete beam and CFRP plate in this paper is modelled with the use of a numerical displacement-based fibber model. A simple approach to the weak zone in the bond description is proposed. Discontinuous bond zone is modelled by modification in the original constitutive law for description of the bond between the reinforced concrete beam and CFRP plate. Verification of implemented modification is analysed using bond stress distribution and tensile plate force distribution along the externally strengthened reinforced concrete beam under equal load for two different cases of discontinuous zone in the bond layer. Numerical analysis carried out for both cases shows that the proposed modelling of the weak zone is appropriate.

## 2. NUMERICAL MODEL FORMULATION

Numerical model, which is used for the analysis of the strengthened beam element, is based on a fibber model [5]. Beam element based on two-node displacement has been used. It has two components: a two-node concrete beam and a strengthening plate. The nodal degrees of freedom of the concrete beam and of the strengthening plate are different to permit slip. The reinforced concrete section is discretized into fibbers layers as shown in Fig. 1.

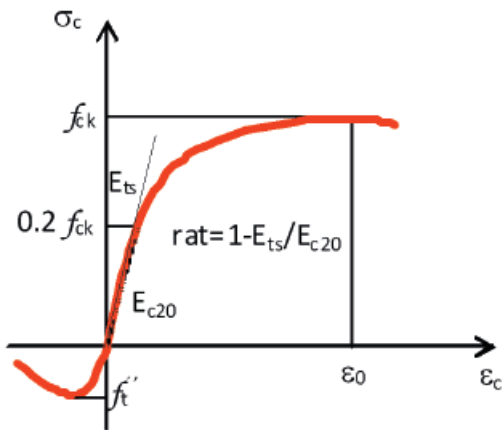
Cubic transverse and linear axial displacement fields are assumed for the beam, and linear axial displacement for the strengthening plate. The distribution of the bond slip is quadratic. The element is implemented in the finite element program FEAP [6].

In the numerical model, the concrete is described by one-dimensional model proposed by Mohd-Yassina [5], while the behaviour of inner and external reinforcement for strengthening are defined by Menegotto-Pinta model [5]. Models for the constitutive laws of concrete and reinforcement are shown in Fig. 2 and Fig 3.



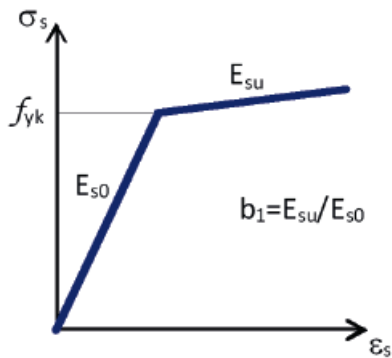
$$U = \{U_1^1 \ U_2^1 \ U_3^1 \ U_4^1 \ | \ U_1^2 \ U_2^2 \ U_3^2 \ U_4^2\}^T \quad u(x) = \{u_B(x) \ v_B(x) \ | \ \bar{u}(x)\}^T$$

Figure 1. Node and field displacement of the reinforced concrete beam model with slip in plate



- $f_{ck}$  compressive strength of the concrete
- $\epsilon_0$  deformation at  $f_{ck}$
- $\epsilon_{cu}$  deformation at 20%  $f_{ck}$
- rat  $1 - E_{ts}/E_{c20}$
- $f_t'$  tensile strength of the concrete
- $E_{ts}$  initial Young's modulus
- $E_c$  concrete Young's modulus at 20%  $f_{ck}$

Figure 2. Concrete constitutive law



- $f_{yk}$  yielding strength
- $E_{s0}$  initial Young's modulus
- $b_1$   $E_{su}/E_{s0}$
- $E_{su}$  modulus of elasticity of the steel after  $f_{yk}$

Figure 3. Reinforcement constitutive law

Constitutive bond law between concrete and external FRP reinforcement is described by the linear relation between displacement and shear stresses in the bond layer up to the maximal bond strength. When this value of bond shear stresses is reached, slip occurs, which means that displacement is increasing while the corresponding shear stress is zero, Fig. 4.

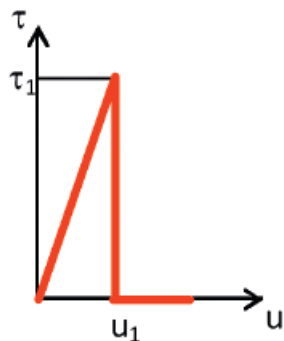


Figure 4. Bond law between concrete and external reinforcement

### 3. ANALYSED CASE

A reinforced concrete beam externally strengthened with CFRP strip is analysed. A 3200 mm beam element, presented in Figure 5, which has 2900 mm span and cross section  $b/h=800/120$  mm is strengthened with CFRP strip with 100 mm width. The strengthened beam is subjected to four-point bending. Due to the symmetry of the case, only a half of the beam is being analyzed.

In this paper two weak bond layers with length of 504 mm and 514 mm have been analysed. Although the weak zones have approximately the same length their starting and ending nodes are different, especially the second one in which the weak zone ends closely to the node where the load acts. Denoting of the discontinuous zones and their start and end nodes are summarized in Table 1.

Table 1. Characteristics of the discontinues zones

Discontinuous zones	Starting node, $i$	$X_{ini}$ [mm]	End node, $j$	$X_{fin}$ [mm]	$\Delta$ [mm]
»d12_30«	12	200	30	704	504
»d19_38«	19	396	38	910	514

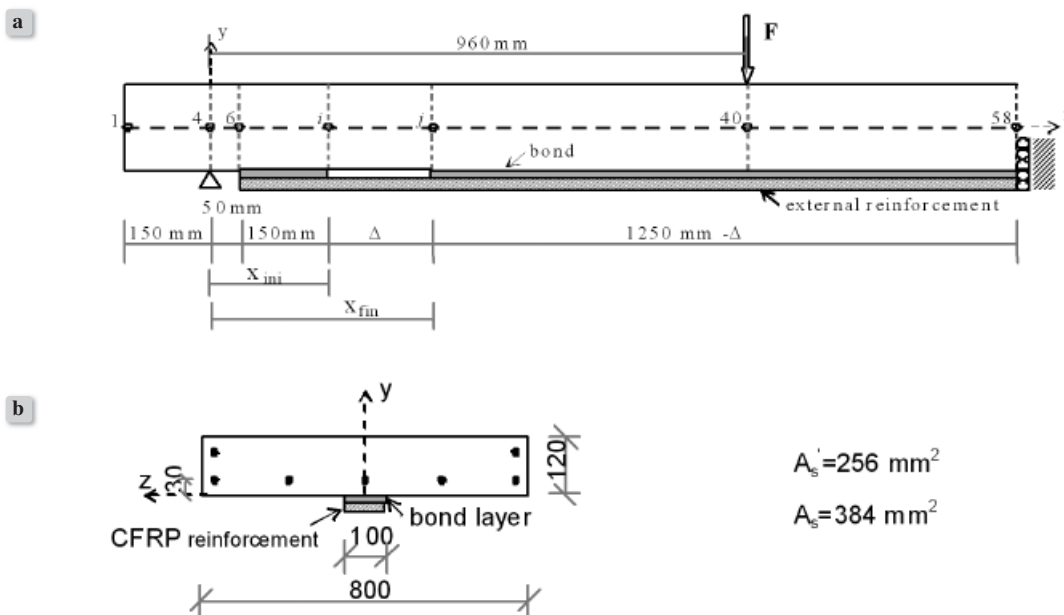


Figure 5. Beam element strengthened with CFRP strip: a) geometry of half beam; b) cross section of the strengthened beam

### 4. MATERIAL PROPERTIES OF THE BEAM AND BOND LAYER BETWEEN CONCRETE AND EXTERNAL CFRP REINFORCEMENT

Values of the parameters for the concrete, inner, as well as external reinforcement used in the analysis are according to the Table 2.

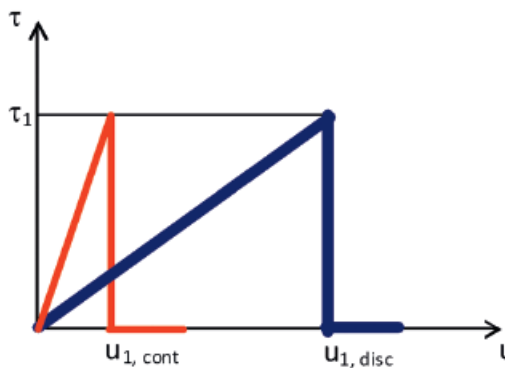
**Table 2.**  
Mechanical properties of the materials:  
a) concrete; b) reinforcement

a					
$f_{ck}$ [MPa]	$\epsilon_0$ [‰]	$\epsilon_{cu}$ [‰]	rat	$f_t'$ [MPa]	$E_{ts}$ [GPa]
-25	-0.0021	-0.01	0.1	1	100

b			
Reinforcement type	$f_{yk}$ [MPa]	$E_{s0}$ [GPa]	$b_1$ [%]
Inner reinforcement	460	210	0.1
External CFRP reinforcement	2400	150	0.1

In order to model weak zone in the bond layer, a modification was introduced in the original constitutive bond law. Maximal shear stress,  $\tau_1$ , remains unchanged, while displacement at slip is significantly increased. By this modification a much more flexible bond is achieved compared to the perfectly bond area. The perfect bond is described by values  $u_{1,cont}=0.0013$  mm and  $\tau_1=3.1$  MPa. Comparison between the original and modified bond law is presented in Figure 6.

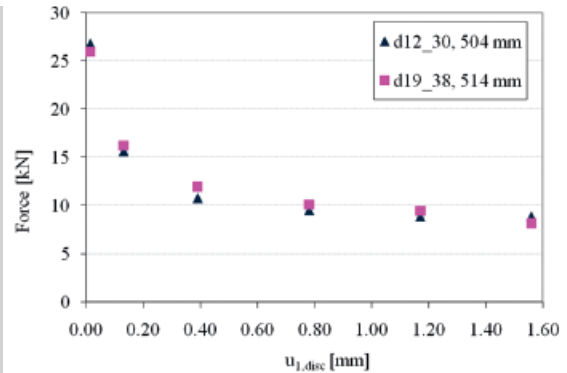


**Figure 6.**  
Comparison between the original and modified bond law

### 5. RESULTS AND DISCUSSION

Introduced modification in the bond law was confirmed by analysis of the numerical model for behavior of discontinuous bond for selected values of parameter  $u_{1,disc}$ .

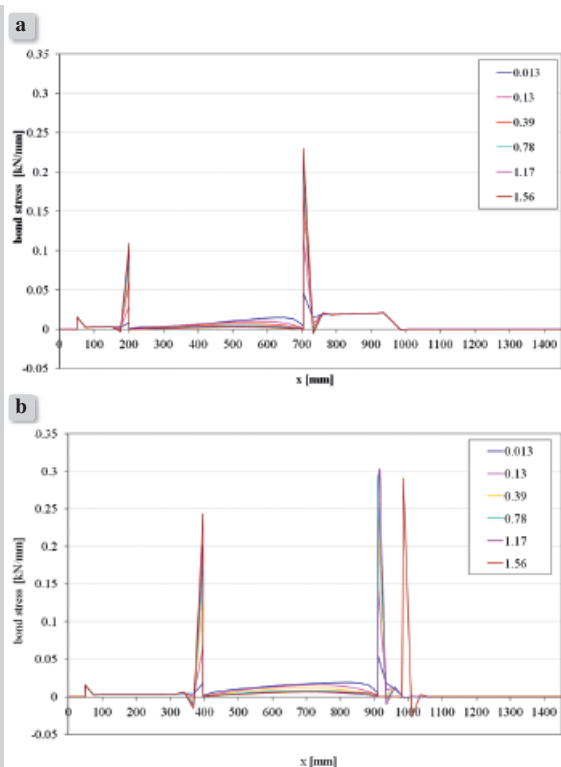
The influence of the selected value for parameter  $u_{1,disc}$  is presented in Figure 7, which has been used in the process of modeling for the two selected discontinuous bonds described in Table 1, on its bearing capacity. For the both discontinuous zones the values of the calculated bearing capacity are decreasing and approaching to an asymptotic value when  $u_{1,disc}$  is increasing. The results of the analysis show that the modification introduced to the model is appropriate.



**Figure 7.**  
Influence of selected value for parameter  $u_{1,disc}$  on bearing capacity

Implemented modification of the constitutive bond law was confirmed by analysis of the bond stress distribution and tensile force in CFRP strip along the beam under equal loading for the two discontinuous zones. Analysis are carried out for selected values of the parameter  $u_{1,disc}$  used in the modeling process of the weak bond zone upon the equal ultimate force of  $F=8.20$  kN.

Figure 8 presents bond stress distribution along the externally strengthened beam for different selected values of the parameter  $u_{1,disc}$  upon the equal ultimate force of  $F=8.20$  kN for the two different discontinuous zones.

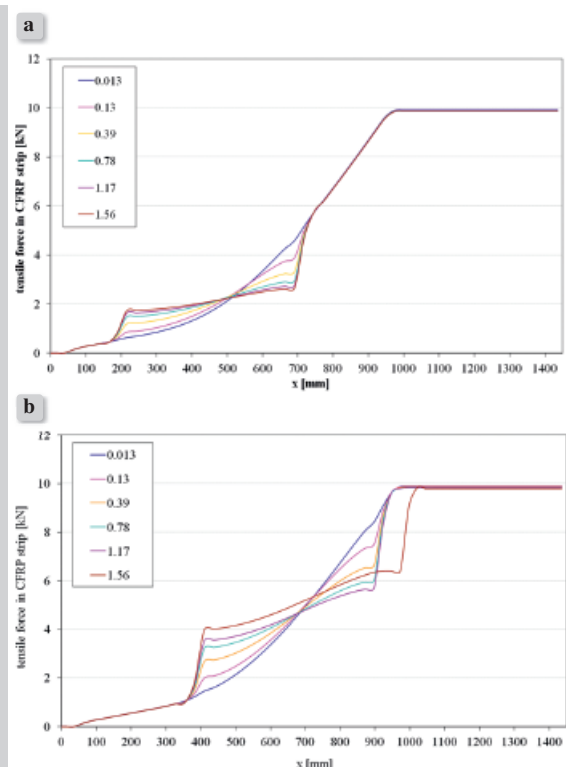


**Figure 8.** Bond stress distribution for selected values of the parameter  $u_{1,disc}$  upon the equal ultimate force of  $F=8.20$  kN: a) discontinuous bond zone »d12\_30«; b) discontinuous bond zone »d19\_38«

By increasing the value of the parameter  $u_{1,disc}$  bond stress increases at the places where cross section changes, at the beginning of the CFRP strip and at the beginning and end of the discontinuous zones. Bond stress distribution in the zones, where the bond between concrete and external CFRP reinforcement is weak, approaches to zero by increasing the value of the parameter  $u_{1,disc}$ .

Should be mentioned that bond stress distribution of the discontinuous zone »d19\_38« modelled with  $u_{1,disc}=1.56$  mm differed from the other bond stress distribution. In that case it comes up to numerical instability marked by bond stress oscillations around the point where the load acts.

Figure 9 presents tensile force distribution in CFRP strip along the beam for different selected values of the parameter  $u_{1,disc}$  used in the modelling process of the weak bond zone upon the equal ultimate force of  $F=8.20$  kN for the two different discontinuous zones.



**Figure 9.** Tensile force distribution in CFRP strip for selected values of the parameter  $u_{1,disc}$  upon the equal ultimate force of  $F=8.20$  kN: a) discontinuous bond zone »d12\_30«; b) discontinuous bond zone »d19\_38«

Based on the Figure 9 one could conclude that in the zone of the perfect bond between concrete and external reinforcement all charts have the same inclination and the same ultimate tensile force in the CFRP strip, which do not depend on the value of the parameter  $u_{1,disc}$  used in the process of modeling. At both ends of the weak zone, tensile force in the CFRP strip is increasing by increasing the value of the parameter  $u_{1,disc}$ . At the zone of the weak bond between concrete and CFRP strip tensile force in the strip approaches a constant value by increasing the value of the parameter  $u_{1,disc}$ .



## 6. CONCLUSIONS

Improper execution of bonding of FRP plates to the RC beam loaded in flexure may lead to the appearance of zones where the bond is substantially weaker, and air pockets are present. The influence of this phenomenon is difficult to be evaluated as it can not be accounted for in the design and planning stage.

This paper presents an attempt for verification of the modification of the numerical model for modeling the bond law between reinforced concrete beam and CFRP strip. A simple approach that consists of a bond constitutive model modification which can be easily incorporated into the existing numerical model is proposed for modelling the weak zones.

The analysis leads to a conclusion that bearing capacity decreases approaching an asymptotic value by increasing the value of the parameter  $u_{1,disc}$ . The results obtained show that the bond stress distribution in the weak zone approaches zero by increasing the value of the parameter  $u_{1,disc}$  used in the process of modeling, which is in accordance with the actual situation. The value of the parameter  $u_{1,disc}$  used in the process of modeling has no influence on the ultimate tensile force in CFRP strip. Tensile force in the CFRP strip in the weak zone approaches constant value by increasing the value of the parameter  $u_{1,disc}$  used in the process of modeling.

The results obtained show that the proposed modification of the bond law is appropriate. Further validation of the proposed model and results obtained by parametric analysis using this model, has to be carried out by the experimental researches.

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