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STATISTICAL MODEL FOR COMPRESSIVE STRENGTH OF LIGHTWEIGHT CONCRETE

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Abstract

The objective of this study is to develop a statistical model of compressive strength of lightweight concrete, using new material test data. The data base includes over 8000 samples, obtained from eight different sources, representing the nominal strength, f_c', from 21 to 50 MPa (3000 to 7000 psi). The presented research is focused on the development of statistical parameters of material properties. Resistance is considered as a product of three random variables representing the uncertainty in material properties, dimensions and geometry (fabrication factor) and analytical model (professional factor). Material test data is presented in form of the cumulative distribution functions (CDF) plotted on the normal probability paper for an easier interpretation of the results. The shape of the CDF is an indication of the type of distribution, and since the resulting CDF's are close to straight lines, they can be considered as normal random variables. In addition, the statistical parameters are determined by fitting a straight line to the lower tail of the CDF. The most important parameters are the mean value, bias factor and the coefficient of variation. It was observed that the quality of material and workmanship has been improved over the last 30 years and this is reflected in reduced coefficients of variation.

Streszczenie

Celem artykulu jest przedstawienie modelu statystycznego wytrzymałości na ściskanie betonu lekkiego, opracowanego na podstawie nowych danych testowych materiału. Baza danych zawiera wyniki testów ponad 8000 próbek, otrzymanych z ośmiu źródeł, reprezentujących nominalną wytrzymałość betonu, f_c', od 21 do 50 MPa (3000 do 7000 psi). Przedstawione badania maja na celu wyprowadzenie parametrów statystycznych właściwości materiału. Wytrzymałość jest rozpatrywana jako iloczyn trzech zmiennych losowych reprezentujących losowość właściwości materiałowych, wymiarów i geometrii oraz modelu analitycznego. Dla ułatwienia interpretacji, wyniki testów zostały przedstawione w formie skumulowanej funkcji rozkładu prawdopodobieństwa (CDF) na znormalizowanym arkuszu probabilistycznym. Kształt CDF określa typ rozkladu i jeśli wykres CDF jest zbliżony do linii prostej, oznacza to, że zmienna losowa ma rozkład normalny. Ponadto parametry statystyczne są określone przez dopasowanie linii prostej do dolnej części krzywej CDF. Najważniejszymi parametrami są wartość średnia, współczynnik odchylenia od wartosci sredniej (bias) i współczynnik zmienności. Zaobserwowano, że jakość materiału i wykonania poprawiła się w ciągu ostatnich 30 lat i znajduje to odzwierciedlenie w zmniejszonym współczynniku zmienności.

Keywords: **Lightweight Concrete (LWC); Compressive strength; Cumulative distribution function; Normal probability paper; Statistical parameters; Resistance factor.**

1. INTRODUCTION

Light weight concrete has been used efficiently for structural purposes for many years. The lightweight concrete mixtures can be designed to achieve similar strength as ordinary concrete but with a reduced mass, and it finds more applications also for the main structural members. The latest trend is the utilization of Self-Consolidating Lightweight High-Performance Concrete in bridge engineering [1]. This requires the development of a rational resistance model for the lightweight concrete members. Statistical analysis of material shows an improved quality and similarity to ordinary concrete. However, additional properties of lightweight concrete have also influence on the members' behaviour in shear and flexure, and this requires a further investigation.

In the limit states design, load effect is compared with load carrying capacity (resistance). The limit state function represents a state of equilibrium, when load and resistance balance each other and the difference between load and resistance is a safety margin. Each failure mode can be represented by a limit state function. The load and resistance parameters can involve a considerable degree of uncertainty and should be treated as random variables. Therefore, reliability is a rational measure of structural performance. The design process, known as the Limit States Design, requires a set of load and resistance factors for each limit state. Objective of the code calibration is to select these factors so that the reliability of designed structures is at the predetermined target level.

This paper documents the derivation of statistical parameters of the compressive strength of lightweight concrete. These parameters showed that the quality of workmanship has improved over the last 20-30 years and this can allow for an increase of resistance factor. The present study is based on a new and representative number of cylinder test results.

Structural lightweight concrete (LWC) has a reduced density compared to normal weight concrete. The density of LWC is in the range of 1440 to 1840 kg/m^3 (90 to 115 pcf) and density of ordinary concrete is in the range of 2240 to 2400 kg/m^3 (140 to 150 pcf). Moreover, LWC mixtures can be designed to achieve similar strength as ordinary concrete. Therefore, structural lightweight concrete provides a more efficient strength-to-weight ratio in structural elements than normal weight concrete. These properties caused that LWC can be used to reduce the dead load of structure, which then allows for reduction of size of structural elements such as columns and footings, and use of less reinforcing steel. Additionally, reduced mass can decrease the risk of earthquake damage to a structure because the earthquake forces are proportional to the mass of structural components [2, 3].

2. MATERIAL TEST DATA

Resistance of a structural component, R, is a function of material properties and dimensions. R is a random variable due to various categories of uncertainties. It is convenient to consider R as a product of three factors,

$$R = R_n \cdot M \cdot F \cdot P \tag{1}$$

where:

R_n – Nominal (design) value of resistance,

M – Material factor representing material properties, in particular strength and modulus of elasticity,

F – Fabrication factor representing dimensions and geometry of the component, including cross-section area, moment of inertia, and section modulus,

P – Professional factor representing the approximations involved in the structural analysis and idealized stress/strain distribution models. The professional factor P is defined as the ratio of the test capacity to analytically predicted capacity. Therefore, P can be considered as a variable representing the model uncertainty.

The statistical parameters for M, F and P were considered by various researchers and the results were summarized by Ellingwood et al. [4], based on material test data available in 1970's. However, it has been observed that the quality of materials such as reinforcing steel and concrete has improved over the years. Therefore, in this study, material test database has been updated. The collection and processing of the new data is one of the major contributions of this study.

The present study is based on the material database provided by the industry.

The tests were performed by producers of materials and submitted in 2009 through the associations representing the industry. Eight different manufacturers of lightweight aggregate submitted the total of 8.889 compressive strength test results for lightweight concretes representing the nominal strengths, f_c', from 21 to 50 MPa. It has to be noted that some of the tests were performed on smaller 100x200 mm cylinders and they are analysed separately from tests performed on standard 150x300 mm cylinders. More explanation about data is presented in a study by Nowak and Rakoczy [5].

The new test data serves as a basis for determining the statistical parameters for material factor, M. It is assumed that the variability of material properties and dimensions correspond to an average quality of construction expected in practice. Long-term changes in concrete and steel affecting strength are not considered.

3. MATERIAL FACTOR

The statistical parameters of material factor, M, are determined from the test data provided by the industry.

The obtained test data are plotted on the normal probability paper. The construction and use of the normal probability paper is described in textbooks [6]. It is a convenient way to present cumulative distribution functions (CDF), as it allows for an easy evaluation of the most important statistical parameters as well as type of the distribution function. The horizontal axis represents the basic variable, i.e. strength of the sample. Vertical axis is the inverse normal probability scale and it represents the distance from the mean value in terms of standard deviations. The vertical coordinate can also be considered as the probability of exceeding the corresponding value of the variable. For any value of concrete strength (horizontal axis), the vertical coordinate of CDF corresponds to a certain probability of being exceeded. For example, value of 1 on the vertical scale corresponds to 0.159 probability that that value of concrete strength will be exceeded.

3.1. Material factor for lightweight concrete

The lightweight concrete database represents the nominal strength, f_c', from 21 to 50 MPa, with densities from 2240 to 2400 kg/m^3. The (CDF) of compressive strength, f_c', are plotted on the normal probability paper in Figure 1 and corresponding statistical parameters of compressive strength f_c' and numbers of samples are listed in Table 1.

3.2. Comparison of LWC and ordinary concrete

For a better presentation of the results, CDF's of the LWC and ordinary concrete are plotted in Figures 2 to 7. The statistical parameters for ordinary concrete are taken from Nowak and Szerszen [7]. For material strength, the most important part of the CDF is the lower tail of CDF. The statistical parameters are summarized in Tables 2 to 7.

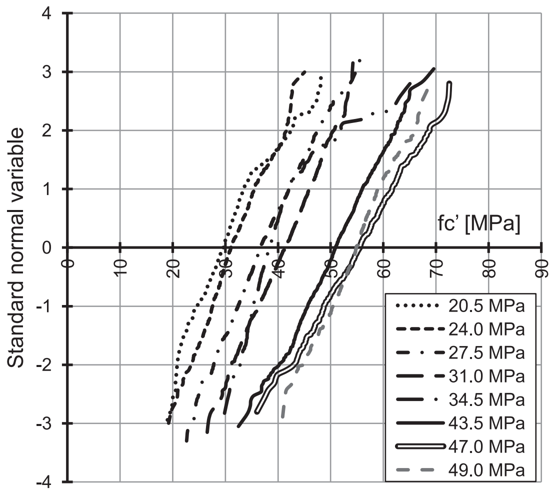

Figure 1.
CDFs for the compressive strength of lightweight concrete (samples 150 x 300 mm)

Table 1.
Statistical parameters for the lightweight concrete (samples 150 x 300 mm)

f_c' (MPa)	Number of samples	Bias factor, λ	Coefficient of variation, V
20.5	609	1.43	0.16
24.0	733	1.3	0.12
27.5	2212	1.34	0.12
31.0	1230	1.33	0.12
34.5	422	1.11	0.08
43.5	876	1.18	0.11
47.0	392	1.19	0.12
49.0	517	1.14	0.10

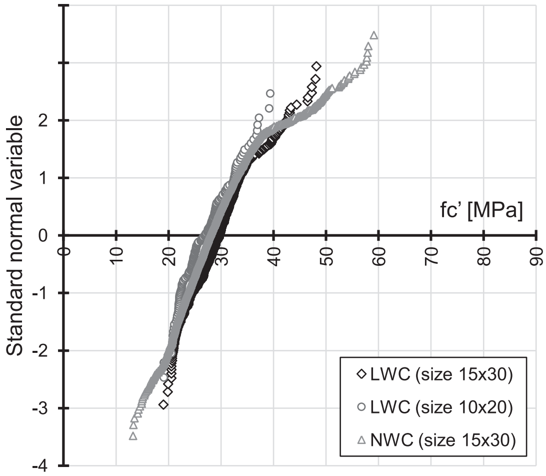

Figure 2.
CDF's of f_c' for LWC 6x12 and 4x8, and ordinary concrete f_c'= 20.5 MPa

Table 2.
Statistical parameters for f_c'= 20.5 MPa

Concrete Samples	Mean value, μ	Bias factor, $\lambda = \mu / f_c'$	Standard deviation, σ	Coefficient of variation, $V = \sigma/\mu$	Number of samples
LWC (size 15x30)	29.6	1.430	4.6	0.160	609
LWC (size 10x20)	25.9	1.250	3.1	0.120	146
Ordinary Concrete	27.6	1.333	4.0	0.145	4104

Table 3.
Statistical parameters for f_c'= 24.0 MPa

Concrete Samples	Mean value, μ	Bias factor, $\lambda = \mu / f_c'$	Standard deviation, σ	Coefficient of variation, $V = \sigma/\mu$	Number of samples
LWC (size 15x30)	**31.3**	**1.296**	**3.8**	**0.122**	**691**
Ordinary Concrete	29.8	1.237	3.5	0.115	552

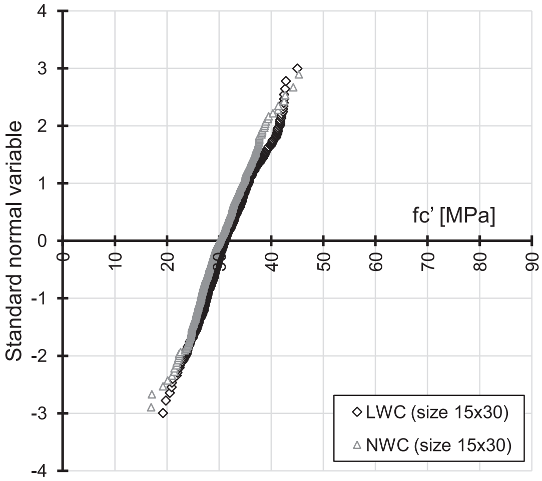

Figure 3.
CDF's of f_c' for LWC 6x12 and ordinary concrete f_c'= 24.0 MPa

For f_c'= 20.5 MPa, the mean value of LWC (150x300 mm) is larger than that for ordinary concrete and the coefficients of variation are almost the same. For size 100x200 mm, the mean value is smaller than for 150x300 mm.

For f_c'= 24.0 MPa (Table 3), the mean value of LWC is larger, compared with the mean value of ordinary concrete. The coefficient of variation of LWC is close to that of ordinary concrete.

For f_c'= 27.5 MPa, the mean value of LWC is larger and the coefficient of variation is smaller compared to ordinary concrete as shown in Table 4. For 100x200 mm, the number of samples is too small for conclusions.

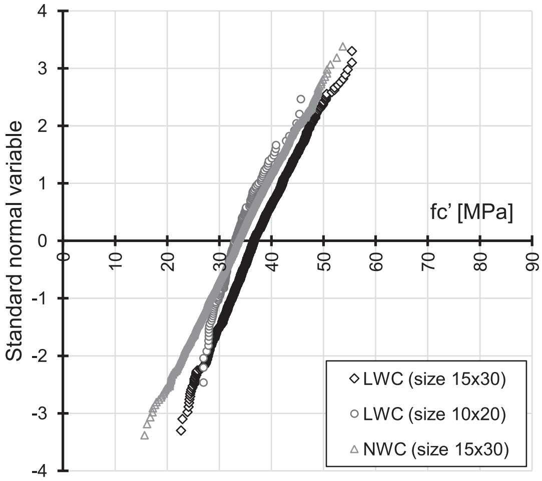

Figure 4.
CDF's of f_c' for LWC 6x12 and 4x8, and ordinary concrete f_c'= 27.5 MPa

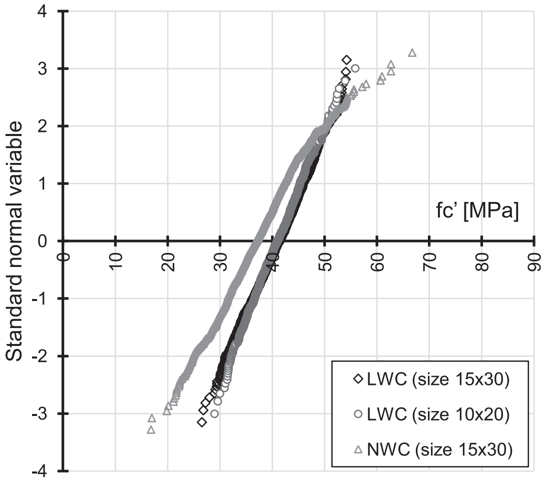

Figure 5.
CDF's of f_c' for LWC 15x30 and 10x20, and ordinary concrete f_c'= 31.0 MPa

Table 4.
Statistical parameters for f_c'= 27.5 MPa

Concrete Samples	Mean value, μ	Bias factor, $\lambda=\mu/f_c'$	Standard deviation, σ	Coefficient of variation, $V=\sigma/\mu$	Number of samples
LWC (size 15x30)	**36.9**	**1.338**	**4.6**	**0.123**	**2067**
LWC (size 10x20)	**33.0**	**1.195**	**3.0**	**0.092**	**145**
Ordinary Concrete	33.4	1.213	5.2	0.155	2900

Table 5.
Statistical parameters for f_c'= 31.0 MPa

Concrete Samples	Mean value, μ	Bias factor, $\lambda=\mu/f_c'$	Standard deviation, σ	Coefficient of variation, $V=\sigma/\mu$	Number of samples
LWC (size 15x30)	**41.2**	**1.328**	**4.8**	**0.117**	**1230**
LWC (size 10x20)	**40.6**	**1.309**	**4.1**	**0.102**	**752**
Ordinary Concrete	36.9	1.189	5.9	0.159	1947

Table 6.
Statistical parameters for f_c'= 34.5 MPa

Concrete Samples	Mean value, μ	Bias factor, $\lambda=\mu/f_c'$	Standard deviation, σ	Coefficient of variation, $V=\sigma/\mu$	Number of samples
LWC (size 15x30)	**38.3**	**1.110**	**2.9**	**0.076**	**422**
LWC (size 10x20)	**48.2**	**1.399**	**5.6**	**0.117**	**483**
Ordinary Concrete	42.4	1.23	5.2	0.122	2082

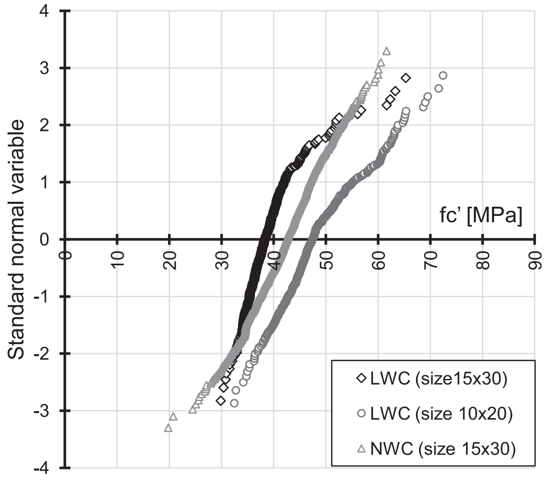

Figure 6.
CDF's of f_c' for LWC 15x30 and 10x20, and ordinary concrete f_c'= 34.5 MPa

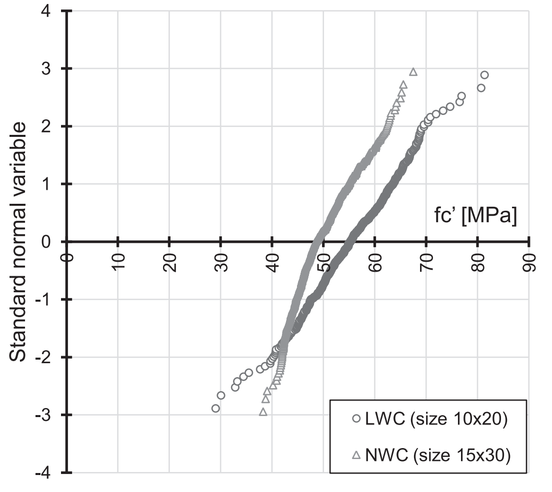

Figure 7.
CDF's of f_c' for LWC 10x20, and ordinary concrete f_c'= 41.5 MPa

For f_c'= 31.0 MPa, the statistical parameters and number of samples are shown in Table 5. The mean value of LWC (15x30 cm) is larger and the coefficient of variation is smaller by about 11%. For 100x200 mm LWC has almost the same parameters as for 150x300 mm.

For f_c'= 34.5 MPa, the number of samples of LWC size 4x8 in. is larger than for 150x300 mm. The mean value of LWC 100x200 mm is larger and the coefficient of variation is similar to the coefficient of variation of ordinary concrete.

Table 7.
Statistical parameters for f_c'= 41.5 MPa

Concrete Samples	Mean value, μ	Bias factor, $\lambda=\mu/f_c'$	Standard deviation, σ	Coefficient of variation, $V=\sigma/\mu$	Number of samples
LWC (size 10x20)	**55.5**	**1.342**	**7.6**	**0.136**	**517**
Ordinary Concrete	48.3	1.167	3.8	0.079	653

Table 8.
Recommended statistical parameters for compressive strength of concrete f_c'

f_c' (MPa)	Compressive strength		Shear Strength	
	Bias factor, λ	**Coefficient of variation, V**	**Bias factor, λ**	**Coefficient of variation, V**
20.5	1.38	0.155	1.38	0.185
24.0	1.33	0.145	1.33	0.175
27.5	1.29	0.140	1.29	0.170
31.0	1.25	0.135	1.25	0.160
34.5	1.22	0.130	1.22	0.155
38.0	1.20	0.125	1.20	0.150
41.5	1.18	0.120	1.18	0.145
45.0	1.16	0.120	1.16	0.145

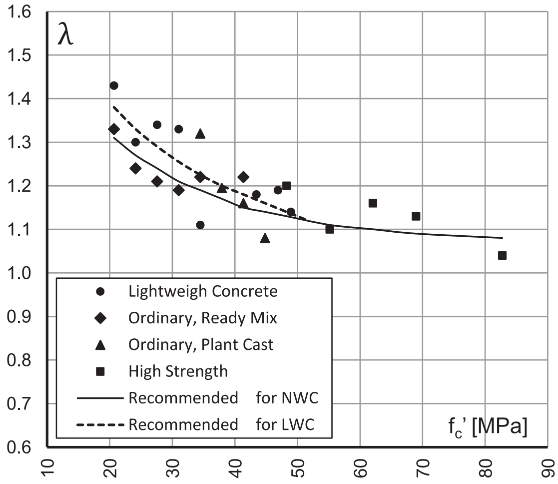

Figure 8.
Recommended bias factors, λ for compressive strength of concrete, f_c'

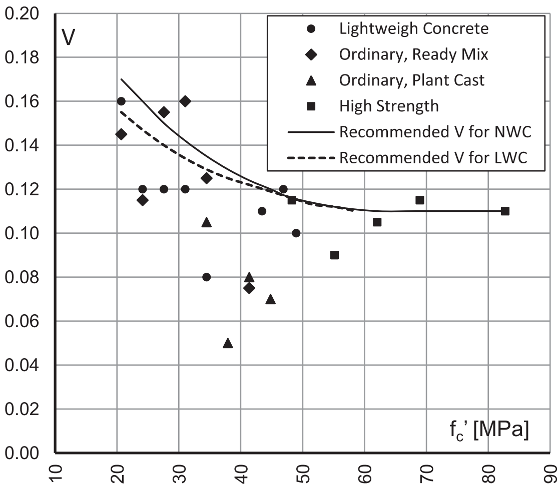

Figure 9.
Recommended coefficient of variation, V, for compressive strength of concrete, f_c'

For f_c'= 41.5 MPa, the statistical parameters and number of samples are shown in Table 7. The data for LWC is only available for size 100x200 mm. The mean value and the coefficient of variation are both larger for LWC.

3.3. Recommended Material Factors for Compressive Strength of Concrete

The test data was analysed to determine the parameters of material factor for calibration of the ACI 318 Code [8]. The bias factor for lightweight concrete varies from λ = 1.43 for f_c' = 20.5 MPa to λ = 1.11 for f_c' = 34.5 MPa.

The results obtained from the test data are presented in Figure 8 for the bias factor and in Figure 9 for the coefficient of variation. Based on the presented findings, it is recommended to use the bias factor and coefficient of variation for compressive strength of concrete as shown in Figure 8 and Figure 9. The bias factors for compressive strength of concrete were determined for the cylinder tests. It has been observed that the strength of concrete in the actual structural component is lower [9]. However, there is

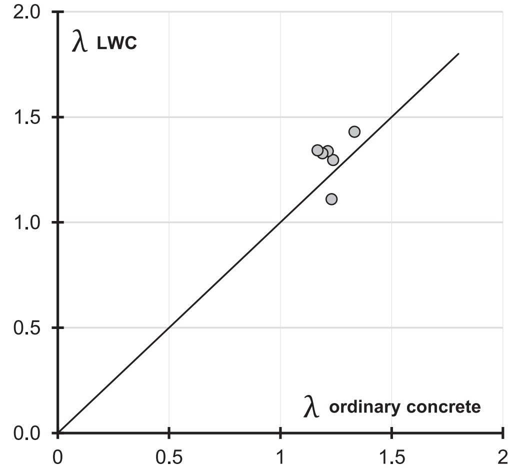

Figure 10.
Bias Factor for LWC and ordinary concrete

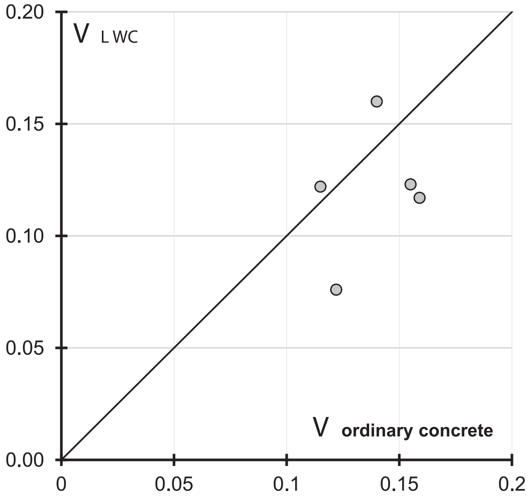

Figure 11.
Coefficient of variation for LWC and ordinary concrete

little data available for quantification of this reduction. The bias factor for shear strength of concrete is assumed to be the same as for compressive strength. The coefficient of variation of shear strength is assumed to be larger than coefficient of variation for corresponding f_c' by 20%. The bias factors and coefficients of variation for concrete strength, selected for the further reliability analysis are summarized in Table 8.

The bias factors for compressive strength of concrete were determined for the cylinder tests. It is assumed that the strength of concrete in the actual structural component is lower. The bias factor for shear strength of concrete is assumed to be the same as for corresponding bias factor for compressive strength. The coefficient of variation of shear strength is assumed to be larger than coefficient of variation for corresponding f_c' by 20%, as shown in Table 8.

4. FURTHER RESEARCH

It was observed that the quality of material and workmanship has been improved over the last 30 years and this is reflected in reduced coefficients of variation of material properties. However, additional properties of lightweight concrete can affect the flexural and shear capacities, and this requires a further investigation. Laboratory tests of members made of lightweight concrete and ordinary concrete show that there is a significant difference between them. It means that the resistance factor for lightweight concrete has to be revised according to statistical model. The new test data served as a basis for determining the statistical parameters for material factor, M. Further research is needed to verify the professional factor, P, which represents the variation in the ratio of the actual resistance and what can be analytically predicted using accurate material strength and dimension values. The updated material and professional factors will serve as a good basis for the reliability analysis and development of new resistance factors for LWC members. New strength reduction factors can be applied in the design Codes as a more efficient way to design LWC members.

5. CONCLUSIONS

The statistical parameters were derived for lightweight concrete based on new material test data provided by industry. The test results were plotted on the normal probability paper, and they confirmed that the quality of material has improved over the last 30 years. It was found that the statistical parameters of compressive strength are slightly better for lightweight concrete compared to that of normal weight concrete. The bias factor for lightweight concrete is slightly higher than that for the ordinary concrete and high strength concrete. Moreover, it was observed that the coefficient of variation of strength of concrete is reduced. This is an indication of a more conservative approach to the application of a relatively new material.

Bias factors for LWC versus ordinary concrete are plotted in Figure 10 and the coefficients of variation in Figure 11.

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