ARCHITECTURE CIVIL ENGINEERING ENVIRONMENT
The Silesian University of Technology No. 2/2011

NUMERICAL STUDY OF FRACTURE PROCESS ZONE WIDTH IN CONCRETE MEMBERS

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Received: 18.04.2011; Revised: 7.06.2011; Accepted: 15.06.2011

Abstract

In the paper the influence of aggregate size on width of fracture process zone w_c is considered. Some researchers observed that the greater grains of aggregate, the wider fracture process zone (FPZ). The average value of the FPZ width taken from tests performed by *Woliński* was 26.6 and it did not depend on maximum aggregate size D_{max}. There are not consistent conclusions as to whether the width of FPZ depends on aggregate size and there are no standard methods of FPZ width measurement.

The problem arises how to choose the width of FPZ in numerical modelling of concrete structures. For example, *Bažant* and *Oh* proposed to take $w_c=3D_{max}$ in numerical calculations. To discus this problem, the authors' own numerical simulations concerning bent concrete member with different width of FPZ: 5, 10, 20, 26.5, 50, 100 mm were performed. On the basis of the comparison of obtained results their significant differences dependent on w_c has been observed. Taking into account the minimum potential energy in a member, it can be said that the most rational thing to do is to take the smallest elongation within the localized microcracking. This condition takes place in the analyzed beam when w_c=50 mm. The assumption $w_c=3D_{max}$ does not fit this criterion. Also the width from the experiment performed by *Woliński* is not in good relation to obtained numerical results.

The main conclusion from this paper is that the width of FPZ does have an influence on obtained numerical results performed by crack band model. The problem of estimating the width of FPZ in numerical simulations exists and requires further research.

Streszczenie

W artykule przedstawiono problematykę wpływu uziarnienia kruszywa na szerokość pasma mikrorys w_c. Wyniki niektórych badań wskazują, że wraz ze wzrostem Dmax szerokość pasma mikrorys zwiększa się. Natomiast średnia szerokość pasma mikrorys uzyskana w badaniach Wolińskiego wyniosła 26.6 i nie zależała od D_{max}. Nie ma natomiast jednoznacznej odpowiedzi odnośnie wpływu uziarnienia kruszywa na szerokość pasma mikrorys oraz brak jest standardowej metody eksperymentalnego wyznaczania w_c.

Powstaje pytanie jak dobierać parametr wc w analizach numerycznych i czy może on mieć wpływ na rezultaty obliczeń. *Bažant i Oh* proponują by w analizach numerycznych przyjmować $w_c=3D_{max}$. Problem ten jest analizowany w artykule. W przeprowadzonych obliczeniach numerycznych dotyczących betonowej belki zginanej przyjęto sześć różnych szerokości pasma mikrorys: 5; 10; 20; 26,5; 50; 100 mm. Dokonano porównania uzyskanych wyników obliczeń i stwierdzono znaczne ich różnice w zależności od przyjętego parametru w_c. Biorąc pod uwagę kryterium minimalnej energii potencjalnej w elemencie, racjonalnym jest przyjęcie najmniejszych wydłużeń w strefie zlokalizowanego zarysowania. Warunek ten osiągnięto w analizowanej belce przy w_c=50 mm. Przyjęcie $w_c=3D_{max}$ nie odpowiada temu kryterium. Również wielkość w_c uzyskana w badaniach Wolińskiego nie ma potwierdzenia w otrzymanych obliczeniach.

W wyniku przeprowadzonej analizy stwierdzono, że szerokość pasma mikrorys przyjmowana w obliczeniach numerycznych ma wpływ na uzyskiwane wyniki. Problem doboru tego parametru przy modelowaniu istnieje i wymaga dalszych badań.

Keywords: **Concrete; Aggregate size; Fracture energy; Width of fracture process zone; Numerical simulation.**

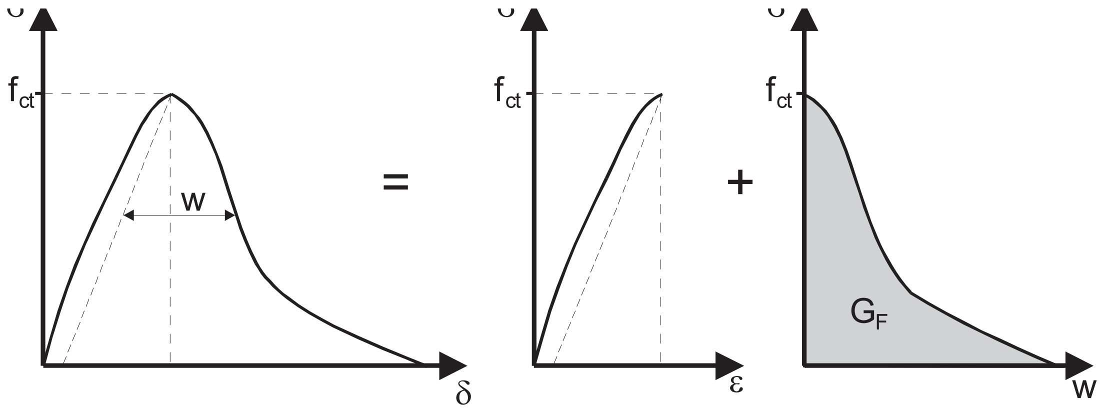

Figure 1.
Geometrical interpretation of concrete fracture parameters

1. INTRODUCTION

Recent advances in nonlinear fracture mechanics give a possibility to analyse crack propagation in concrete structures. There are two ways of modelling cracking using a finite element analysis. In the first concept crack is considered as densely distributed throughout the finite area of element. The alternative approach assumes an isolated sharp interelement crack. The first concept of smeared crack has practical advantages and is mostly used in numerical computations.

In fracture model proposed by *Hillerborg* [1], concrete fracture properties are characterized by three main parameters – axial tensile strength (f_{ct}), fracture energy (G_F) and shape of the stress-deformation diagram given by two curves: stress-strain (σ-ε) and stress-crack opening curve (σ-w – see Fig. 1. The decrease in stress under increasing deformation is called strain softening and it takes place in the narrow zone where the progressive microcracking appears – see Fig. 2. The width of the microcrack band, which is called the width of fracture process zone (w_c), is the additional parameter taken into account when fracture in concrete is modelled as a smeared crack band. There are different opinions about this parameter. Sometimes it is treated as concrete property but sometimes it is assumed to be dependent on size, geometry and static scheme of structural member.

In practice tensile concrete strength is determined in splitting tensile test and fracture energy in three-point bend test but there is no standard method of FPZ measurement. The values of wc obtained during experiments vary significantly due to the use of different type of tests and different shape and volume of tested specimens [2]. Some experiments on standardization of FPZ width test have been performed, for example [3, 4], yet they have not led to finalization. The question is how to take FPZ width in numerical computations of concrete members using the crack band model. This problem is discussed in this paper.

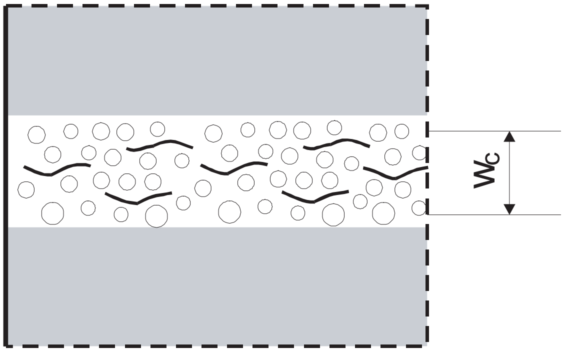

Figure 2.
The width of fracture p

2. THE STUDY OF FPZ WIDTH ESTIMATION

As there are no standard methods to determine the width of FPZ experimentally, it may be approximately calculated from the equation:

$$w_c = \frac{2G_F}{f_{ct}^2}\left(\frac{1}{E_c}-\frac{1}{E_t}\right)^{-1},$$

where E_c is elastic modulus of concrete, and E_t is tangent softening modulus of a declining segment of the stress-strain diagram. This is just an effective width corresponding to linear stress-strain diagram and to assumed uniform strain distribution within the fracture process zone.

Some authors point out that w_c is an independent material parameter, which can differ from concrete to concrete, and it depends on D_{max}. The ratio of w_c to D_{max}, presented in professional technical literature [5], ranges from 1.0 to 5.0 for various kinds of concrete.

Bažant, Oh [5] came to a conclusion that the boundary of the localized cracking region should not be limited only as the boundary of visible microcracks but as the boundary of the whole strain-softening region. In their opinion it is generally possible to assume, in practical cases, that the optimum width of FPZ is about three-times the maximum aggregate size. The possible reason for the influence of aggregate graining on the value of fracture energy and the width of FPZ, given by *Hu, Duan* [6], is the non-uniform distribution of local fracture energy. The presence of large size aggregates prevents the crack from opening and results in wider FPZ.

Interesting experiments were performed by *Otsuka, Date* [7]. When comparing fracture process zone traced from X-ray films, they observed a significant influence of aggregate size on the width of the microcrack zone. The results obtained by acoustic emission technique showed the relationship between the width and the length of FPZ. With the increase of maximum aggregate size, the width of FPZ increased whereas the length of FPZ decreased. From Mihashi, Nomura tests [8] it was found that the length of FPZ was independent of the heterogeneity but the width was obviously influenced by the aggregate size.

Quite different experimental results were obtained in tests performed by *Woliński* [9]. He did not find marked relationships between the fracture concrete parameters and maximum aggregate size. The obtained mean value of the width of FPZ was 26.6 mm and it did not depend on D_{max}.

Some researchers, for example [10, 11], observed that the dimensions of FPZ were greatly influenced by the specimen size. The size effect on fracture properties of concrete was broadly described by *Bažant, Planas* [12].

There are no consistent conclusions as to whether the width of FPZ depends on the maximum aggregate size. The task of standardizing the testing procedure and the method of estimating the width of fracture process zone has not been undertaken yet. Therefore, there are difficulties with performing numerical simulations of concrete structures based on crack band model of nonlinear fracture mechanics in which it is necessary to model the width of FPZ. The question arises as to how the choice of the width of fracture process zone influences the results of numerical calculations. To analyze this problem, the own numerical simulation was performed in case of concrete beams.

3. NUMERICAL SIMULATIONS

The numerical calculations were performed using the commercial FEM program ALGOR. A concrete beam was computed with the rectangular cross section and the following dimensions: width $b = 0.15$ m, height $h = 0.30$ m, total length $L = 3.00$ m, span $l = 2.70$ m. The specimen was unnotched. The four-point bend test was chosen for the simulations. The beam was loaded symmetrically by two concentrated forces, which were applied from bottom towards the top.

The FEM-analysis was performed on one half of the concrete beam since the four-point test is symmetrical. The FEM-beam was made by three-dimensional brick elements and truss elements. The brick elements were used in the bulk material behind the fracture process zone and truss elements were used only in FPZ. The dimensions of brick elements in the support zone and in the region of the crack were twice as small as in the rest of the beam. The assumed FEM-mesh allowed to obtain nonlinear stress distribution in FPZ. The FPZ was modelled in the region of the biggest bending moment. The biggest values of a bending moment were obtained in the sections of the forces applying due to the fact of the reversed load scheme and the influence of the weight of the beam. To analyze the influence of the FPZ width on the numerical calculations results different widths were taken for modelling this zone: $w_c = 5; 10; 20; 26.5; 50$ and 100 mm. Two of chosen values of w_c are characteristic, $w_c = 26.5$ mm – the value of the crack band width obtained experimentally by *Woliński* [9] and $w_c = 100$ mm – the value equaling three times the maximum aggregate size as it was proposed by *Bažant*, *Oh* [5]. The finite element mesh for the analyzed beam in case of $w_c = 10$ mm is shown in Fig. 3.

While performing FEM-calculations, the following material properties were used:

– the tensile strength $f_{ct} = 1.5$ MPa;
– the compressive strength $f_c = 20.5$ MPa;
– the modulus of elasticity $E_{cm} = 22$ GPa;
– the fracture energy $G_F = 83$ Nm/m^2;
– the maximum size of aggregate $D_{max} = 32$ mm.

In the region of the fracture zone the concrete was modelled as nonlinear material whereas outside this zone it was modelled as linear elastic one. To describe concrete in the fracture region, the tensile concrete model given in CEB-FIP Model Code [13] was used (see Fig. 4). It has been shown in [14], that this model is suitable for FEM analysis of cracking in flexural members.

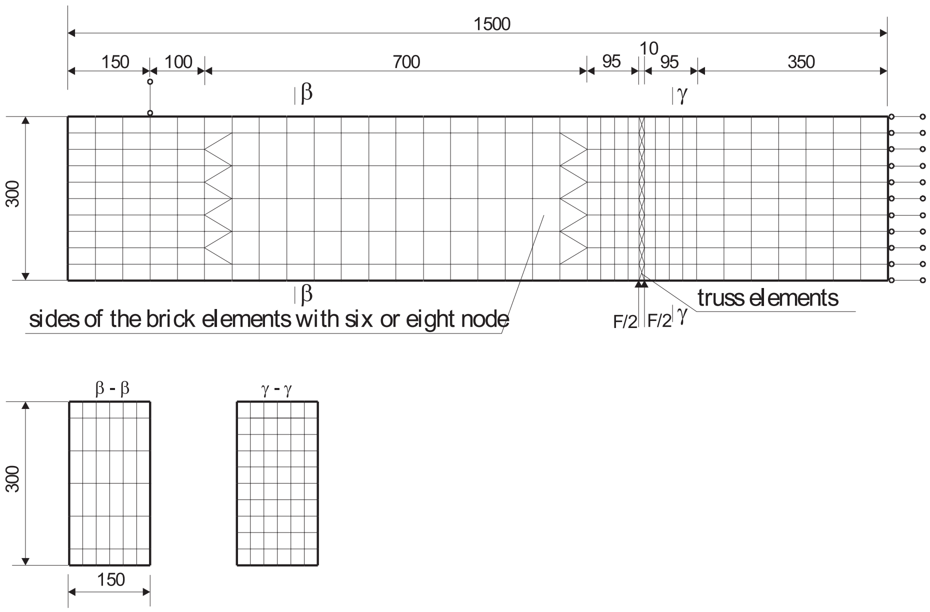

Figure 3.
The FEM-mesh for a beam with wc=10 mm

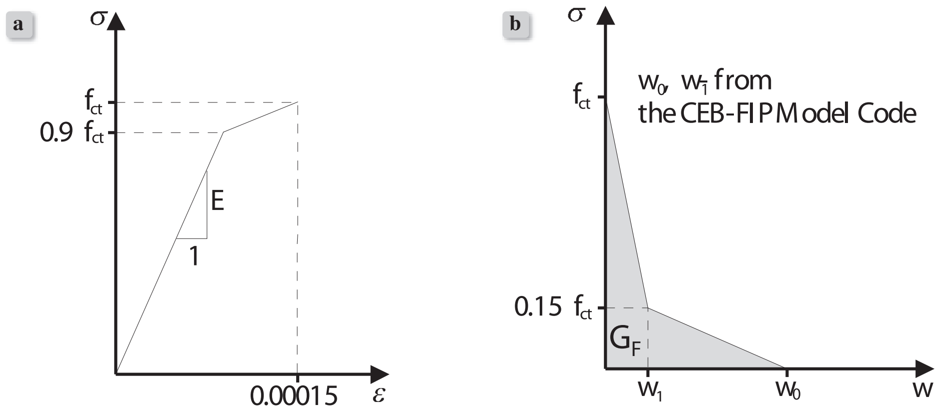

Figure 4.
Diagrams recommended by CEB [13] for the tensile zone of concrete: (a)stress versus strain, (b) stress versus crack opening

4. ANALYSIS OF NUMERICAL RESULTS

As a result of the numerical simulations, the dislocations of nodes and stress components along three axes of the global coordinate system were obtained. On the basis of a comparison of numerical results for six computed concrete beams with different width of FPZ used in calculations, the influence of the FPZ width on obtained numerical results is discussed below.

Firstly, the elongation on the base 250 mm long was calculated in all beams with different width of FPZ. The base was situated in the tensile zone where the crack was modelled. The results of the calculations in succeeding load stages were compared and they are presented in Fig. 5. Furthermore, diagrams of normal stress distribution along the height of the cross section in FPZ for analyzed beams were made. The obtained normal stress diagrams are juxtaposed in Fig. 6 for concrete beams with different w_c.

When analyzing the diagrams presented in Fig. 5 and Fig. 6, the differences in calculation results when compared with the FPZ width used in FEM-calculations are noted. In Fig. 5 we can observe that greater concrete elongations were obtained in cases of the modelled beams where w_c was more than 20 mm. Comparing the stress distributions presented in Fig. 6, we can notice that the greater the width of fracture zone, used in FEM-calculation, the less intensive strain softening of tension concrete and the crack formed more slowly. The confrontation of obtained results presented in Fig. 5 and Fig. 6 points

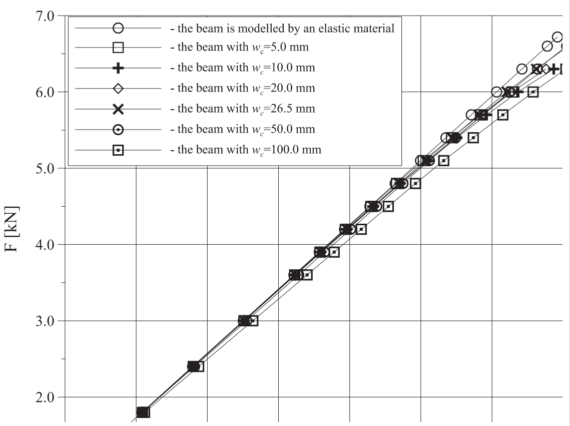

Figure 5.
Comparison of the elongation for beams with different w_C

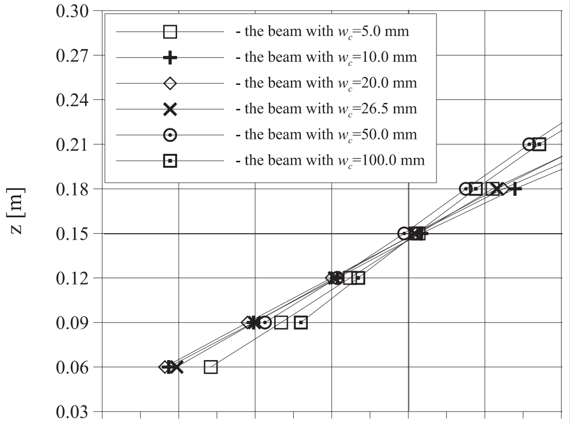

Figure 6.
Comparison of normal stress distribution along the fracture zone in beams with different width wc at the same load stage F = 6 kN

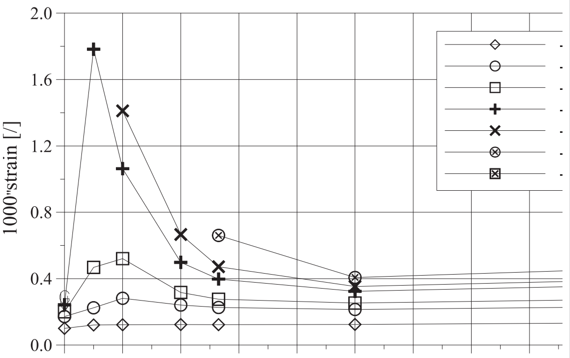

Figure 7.
Concrete strain within the fracture zone at different wc

that a choice of the width of FPZ is an important parameter in numerical modelling of concrete cracking.

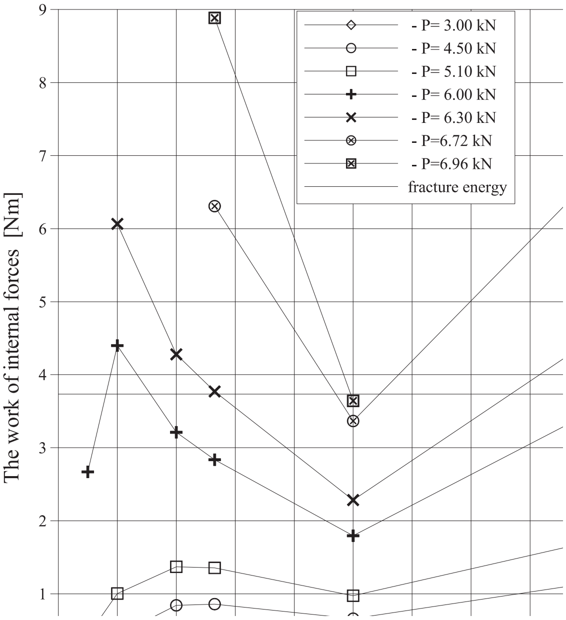

Figure 8.
The work of internal forces

In order to analyze the influence of the fracture zone width on numerical results more precisely, concrete strains within the modeled FPZ and the work of internal forces were calculated (see Fig. 7 and Fig. 8).

On the basis of concrete strains in fracture zone presented in Fig. 7 we can see a significant influence of w_c on obtained results. At the beginning load levels, when $F = 3.0$ and 4.5 kN, strain values were similar. At higher load levels concrete strains were much greater in beams with $w_c = 5$ and 10 mm compared with other beams. Although differences of concrete strains for beams with wc $w_c = 26.5$, 50 and 100 mm were not significant but it may be noticed that the smallest strain was reached in the beam with $w_c = 50$ mm. Big differences in obtained numerical results can be observed when comparing the work of internal forces in beams with different fracture process zone (see Fig. 8). Here we can see a deep "valley" in obtained values. The smallest work of internal forces was reached for the beam with $w_c = 50$ mm. It is worth to notice that only for the beam with the modelled fracture zone 50 mm the work of internal forces did not exceed the fracture energy.

Taking into account the minimum potential energy in a member, it may be said that the most rational thing to do is to take the smallest elongation within the localized microcracking where the crack appears. In analyzed beams this condition takes place when w_c is 50 mm. If we take $w_c = 3D_{max}$ as proposed in the literature [5] (in analyzed beam it would be $w_c = 100$ mm, because $D_{max} = 32$ mm) such an assumption does not fit this criterion. Also the width from the experiment performed by *Woliński* [9], $w_c = 26.6$ mm, is not in good relation to obtained FEM-calculation results.

5. CONCLUSIONS

As the width of FPZ is the additional parameter described in crack band model it should be properly applied in numerical simulations of concrete structures. The adequate choice of this parameter during the numerical calculation is a condition of obtaining correct results performed by finite element method. The numerical analysis presented in the paper shows that the width of FPZ is an important fracture parameter of concrete which has an influence on the FEM-results. Significant differences in obtained results which were observed especially in concrete strain within the microcracked zone and in the work of internal forces prove that this parameter has an influence on FEM-results.

There are no definite conclusions as far as the influence of aggregate size on fracture process zone dimensions is concerned and there are no rules how to determine a width of microcracked zone. The performed numerical simulations were not wide enough to conclude about the relation between maximum aggregate size and width of FPZ. In case of the calculated beam, the obtained numerical results did not confirm the *Bažant* and *Oh* proposition to use $w_c = 3D_{max}$ and they were not in good relation to Woliński experiment in which $w_c = 26.6$ mm.

The main conclusion from the performed analysis is that the width of FPZ does have an influence on obtained numerical results performed by crack band model. The problem of estimating the width of FPZ in numerical simulations exists and requires further research. The authors plan to deal with this problem in future, in particular to elaborate the FEM-results dependence on the scale effect and the type of FEM-mesh used in numerical simulations.

ACKNOWLEDGEMENT

The problem discussed in this paper was presented during the Ninth International Symposium on Brittle Matrix Composites (BMC9) held in Warsaw, 25-28 October, 2009.

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