A R C H I T E C T U R E C I V I L E N G I N E E R I N G E N V I R O N M E N T
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SHEAR RESISTANCE OF REINFORCED CONCRETE BEAMS WITHOUT WEB REINFORCEMENT

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Abstract

The objective of this paper is to review the available data base and shear models for reinforced concrete beams without web reinforcement and select the most efficient model for the design code for concrete structures. The test results are used to establish the relationship between the shear capacity and parameters such as width and depth of the beam cross-section, longitudinal reinforcement ratio, and compressive strength of concrete. Five shear analysis models are considered: ACI 318 Sectional Shear Design Provisions Eq. 11-3 and Eq. 11-5 [1], Equation by Frosch [2], Equation by Zsutty [3], and Equation by the Eurocode 2 [4]. The best fit for the test data is provided by Zsutty's Equation. However, taking into consideration accuracy, required input data and simplicity, Frosch's equation could be recommended for the design code.

Streszczenie

Przedmiotem artykułu jest przegląd dostępnych danych i modeli ścinania dla żelbetowych belek bez zbrojenia poprzecznego oraz wybór najbardziej odpowiedniego modelu do przepisów projektowania konstrukcji żelbetowych. Wyniki badań zostały wykorzystane do ustalenia zależności między nośnością ścinania a takimi parametrami, jak szerokość i wysokość przekroju belki, stopień zbrojenia podłużnego, czy wytrzymałość betonu na ściskanie. Pięć modeli do analizy ścinania wzięto pod uwagę: ACI 318-95 [1] – równania 11-3 i 11-5; równanie Froscha (2003) [2], równanie Zsutty'ego (1968) [3] oraz równania zawarte w Eurokodzie 2 (2004) [4]. Stwierdzono, że najlepszą zgodność z wynikami badań wykazuje równanie Zsutty'ego. Jednakże, rozważając dokładność, wymagane dane na wejściu i prostotę, równanie Froscha można zalecić do przepisów projektowania.

Keywords: **Concrete structures; Reinforced-concrete structures; Shear resistance; Models for shear.**

1. INTRODUCTION

Shear resistance of reinforced concrete beams has been a subject of intensive research for over 50 years. Nevertheless, there is still a disagreement between the researchers as to which model is the most appropriate. In the last decade many approaches were developed including the Modified Compression Field Theory and Strut and Tie Methods. These methods provide consistent and reliable prediction of the ultimate resistance, though, they are complex and time-consuming in use.

Traditional ACI 318 [1] equations 11-3 and 11-5 were developed in 1950's based on the database of 430 specimens [5]. Most recent data indicate that in some cases, a traditional design approach can be too permissive, in particular for members with deep cross-sections, high concrete strengths, or high stress levels in the longitudinal reinforcement. Therefore, ACI requires at least the minimum shear reinforcement when factored shear force due to loads, V_u, exceeds 0.5 ϕV_c, where V_c is the shear capacity of concrete and ϕ is the resistance factor.

2. SCOPE OF THE STUDY

Analysis is based on the database of over 200 shear test results [6-30] used by Frosh [2]. This database includes tests with a wide range of reinforcement ratios, ρ (Table 1), shear span (Table 2), concrete strength (Table 3) and section depth (Table 4). The collected data is for simply supported beams, rectangular in cross-section with shear span equal to or exceeding 2.5d, where d is the depth of the beam.

Table 1.
Number of samples for different longitudinal reinforcement ratios

Longitudinal reinforcement ratio	Number of samples
<1%	24
1.0% - 1.5 %	23
1.5% - 2.0 %	48
2.0% - 2.5 %	30
2.5% - 3.0 %	16
3.0% - 4.0 %	37
4.0% - 5.0 %	46
5.0% -7.0%	5

Table 2.
Number of samples for different compressive strength of concrete

f_c [ksi]*	Number of samples
< 2	9
$2 \leq f_c < 3$	35
$3 \leq f_c < 4$	60
$4 \leq f_c < 5$	40
$5 \leq f_c < 6$	40
$6 \leq f_c < 9$	26
$9 \leq f_c < 13$	19

*f_c [ksi] = 6.895 MPa

Table 3.
Number of samples for different shear spans

Shear span (a/d)	Number of samples
$2.5 \leq a/d < 3$	51
$3 \leq a/d < 4$	104
$4 \leq a/d < 5$	57
$5 \leq a/d < 7.3$	17

Table 4.
Number of samples for different beam depth

Section depth [cm]	Number of samples
$10 \leq h < 25$	9
$25 \leq h < 50$	204
$50 \leq h < 100$	16

3. SHEAR RESISTANCE MODELS

The statistical analysis is performed for the following five equations that are available for calculation of shear resistance of reinforced concrete beams without web reinforcement:

– Equation 11-3 by ACI 318 [1]
– Equation 11-5 by ACI 318 [1]
– Equation by Frosch (2003) [2]
– Equation by Zsutty (1968) [3]
– Equation by by Eurocode 2 [4]

3.1. Shear Design by ACI 318 [1]

According to ACI 318 Code [1] shear strength provided by concrete for non-prestressed members subjected to shear and flexure can be calculated from the following equation:

$$V_c = 2\sqrt{f_c'}b_w d\,, \quad \text{(US units)}, \tag{1}$$

$$V_c = 0.17\sqrt{f_c'}b_w d\,, \quad \text{(SI units)}, \tag{1a}$$

where:

f'_c – compressive strength of concrete, psi (1 psi = 6.895 Pa),

b_w,d – width and depth of effective cross section, in (1 inch = 25 mm).

Although, the tests confirmed that shear strength is affected by the main reinforcement, it is completely neglected in this equation.

ACI 318 [1] also provides a much more complex equation:

$$V_c = \left(1.9\sqrt{f'_c} + 2500\rho_w \frac{V_u d}{M_u}\right) b_w d \le 3.5\sqrt{f'_c}\, b_w d \text{, (US units)}, \quad (2)$$

$$V_c = \left(0.16\sqrt{f'_c} + 17\rho_w \frac{V_u d}{M_u}\right) b_w d \le 0.29\sqrt{f'_c}\, b_w d \text{, (SI units)}, \quad (2a)$$

M_u,V_u – factored moment and factored shear force at the cross-section, kips and kips-ft (1 kip = 4.5 kN, 1 ft = 300 mm),

ρ_w - longitudinal reinforcement ratio.

3.2. Shear Design by Frosch [2]

Frosch (2003) proposed an formula for calculating concrete contribution to shear strength. The following equation is also applicable for steel and FRP–reinforced beams:

$$V_c = 5\sqrt{f'_c}\, b_w d\left(\sqrt{2\rho\frac{29000}{57\sqrt{f'_c}} + \rho^2\left(\frac{29000}{57\sqrt{f'_c}}\right)^2} - \rho\frac{29000}{57\sqrt{f'_c}}\right) \quad (3)$$

$$= 0.4\sqrt{f'_c}\, b_w d\left(\sqrt{2\rho\frac{205}{4.62\sqrt{f'_c}} + \rho^2\left(\frac{205}{4.62\sqrt{f'_c}}\right)^2} - \rho\frac{205}{4.62\sqrt{f'_c}}\right) \quad (3a)$$

f'_c – compressive strength of concrete, psi (1 psi = 6.895 Pa),

b_w,d – width and depth of effective cross section, in (1 inch = 25 mm),

ρ_w – longitudinal reinforcement ratio.

3.3. Shear Design by Zsutty [3]

Zsutty (1968) used a combination of dimensional analysis and statistical regression to obtain an empirical equation:

$$V_c = 59 \cdot \sqrt[3]{\frac{f'_c \rho_w d}{a}}\, b_w d \text{, (US units)}, \quad (4)$$

$$V_c = 2.138 \cdot \sqrt[3]{\frac{f'_c \rho_w d}{a}}\, b_w d \text{, (SI units)}, \quad (4a)$$

for members with $\frac{a}{d} > 2.5$. When $\frac{a}{d} \le 2.5$ to account for arching action, he proposed to use an additional multiplier:

$$V_c = 59 \cdot \sqrt[3]{\frac{f'_c \rho_w d}{a}}\, b_w d \text{, (US units)}, \quad (5)$$

$$V_c = 2.138 \cdot \sqrt[3]{\frac{f'_c \rho_w d}{a}}\, b_w d \text{, (SI units)}, \quad (5a)$$

where:

f'_c – compressive strength of concrete, psi (1 psi = 6.895 Pa),

b_w,d – width and depth of effective cross section, in (1 inch = 25 mm),

ρ – longitudinal reinforcement ratio,

a – shear span.

3.4. Shear Design by Eurocode 2 [4]

A design equation specified by the Eurocode 2 (2004) for shear resistance, $V_{Rd,c}$, of the members without shear reinforcement is as follows:

$$V_{Rd,c} = \left[C_{Rd,c} k (100\rho_l f_{ck})^{\frac{1}{3}} + k_1 \sigma_{cp}\right] b_w d \ge (v_{min} + k_1 \sigma_{cp}) b_w d \quad (6)$$

where

f_{ck} – characteristic value of concrete compressive strength, MPa,

$k = 1 + \sqrt{\frac{200}{d}} \le 2$ – with d [mm],

$\rho_l = \frac{A_{sl}}{b_w d} \le 0.02$ – ratio of longitudinal reinforcement,

A_{sl} – area of longitudinal reinforcement,

b_w – the smallest width of cross-section in the tensile area [mm],

$\sigma_{cp} = \frac{N_{Ed}}{A_c} < 0.2 f_{cd}$ – stress due to the axial force [MPa],

f_{cd} – design value of concrete compressive strength,

A_c – area of concrete cross-section [mm²],

$C_{Rd,c} = \frac{0.18}{\gamma_c}$

$k_1 = 0.15$ – recommended value,

$v_{\min} = 0.035 k^{\frac{3}{2}} \cdot f_{ck}^{\frac{1}{2}}$

$\gamma_c = 1.5$ – partial safety factor for concrete,

4. STATISTICAL ANALYSIS

The considered shear equations were compared to the test data. For each data point, the value obtained from the test was divided by the calculated shear capacity. The obtained ratios are plotted on the normal probability paper. The construction and use of the normal probability paper can be found in the textbook on probability (e.g. Nowak and Collins [6]).

The Cumulative Distribution Function (CDF), of the ratio, V_{test}/V_{calc}, where V_{calc} is the shear capacity calculated using the considered equations. The resulting CDF was obtained by plotting the pairs of coordinates x_i, y_i where:

$$z_i = \phi^{-1}(p_i) \tag{7}$$

$$p_i = \frac{i}{N+1} \tag{8}$$

where

x_i – ratio of V_{test}/V_{calc}, arranged in a non-decreasing order, $i = 1, 2,..N$,

N – number of samples.

The CDF was then approximated by a straight line that represents a normal distribution. The statistical parameters of Vtest/Vcalc (treated as a random variable) were obtained directly from the graph: the mean value corresponds to z = 0 and the mean value plus standard deviation corresponds to z = 1.

Experimental data was sorted so that it was possible to determine the relationship between the bias factor and longitudinal reinforcement ratio, shear span, and concrete compressive strength. Figures 1 to 5 present the CDF's of the bias factor for resistance for different longitudinal reinforcement ratios. Figure 1 indicates that equation 11-3 is very sensitive with regard to longitudinal reinforcement ratio. The mean value depends on the reinforcement ratio. However, the standard deviation is comparable for all sets of data. Frosh formula shown in Figure 3 seems to be more conservative as the mean value is about 1.25 and lower tail of CDF approaches 1.

Figures 6 to 10 present CDF's for different shear span to effective depth ratios. As in other cases, the ACI equation 11-3 produces the largest variation. CDF's corresponding to the remaining equations have the same slope and lower value of standard deviation and a very small variation of the mean values for different shear span ratios.

Figures 11 to 15 show CDF's for different concrete compressive strengths. Figure 15 indicates that the equation used in Eurocode 2 (2004) accurately includes the concrete strength as there is almost no variation in the mean values.

Figures 16 to 18 present coefficients of variations calculated for different longitudinal reinforcement ratios, span ratio and concrete compressive strength, respectively. It is clearly seen that all the equations, excluding ACI 318 Equation 11-3, show a similar degree of variation.

Table 5 presents the statistical parameters calculated for a set of all data for all the equations. Excluding ACI 318 [1] equation 11-3, all the formulas have comparable statistical parameters. Frosch [2] formula has a slightly larger standard deviation but at the same time it is more conservative in comparison to others as the mean value is about 1.3.

5. CONCLUSIONS

The considered equations differ significantly with respect to their form, derivation and applicability. Study showed that ACI 318 [1] equation 11-3 has a significant variation. It is very sensitive to all the shear resistance parameters. However, due to its simplicity it is most commonly used for calculation of shear capacity of the beams. There is a need for a formula that can replace ACI 318 equation 11-3, that can provide more reliable results. Frosch [2] formula seems to be a good one. The most significant advantage of this equation is its simplicity. To calculate the shear resistance, a designer is required to provide only four basic parameters: concrete compressive strength, beam width, effective depth, and longitudinal reinforcement ratio. Despite its simplicity Frosch equation provides consistent results compared to other much more complex equations.

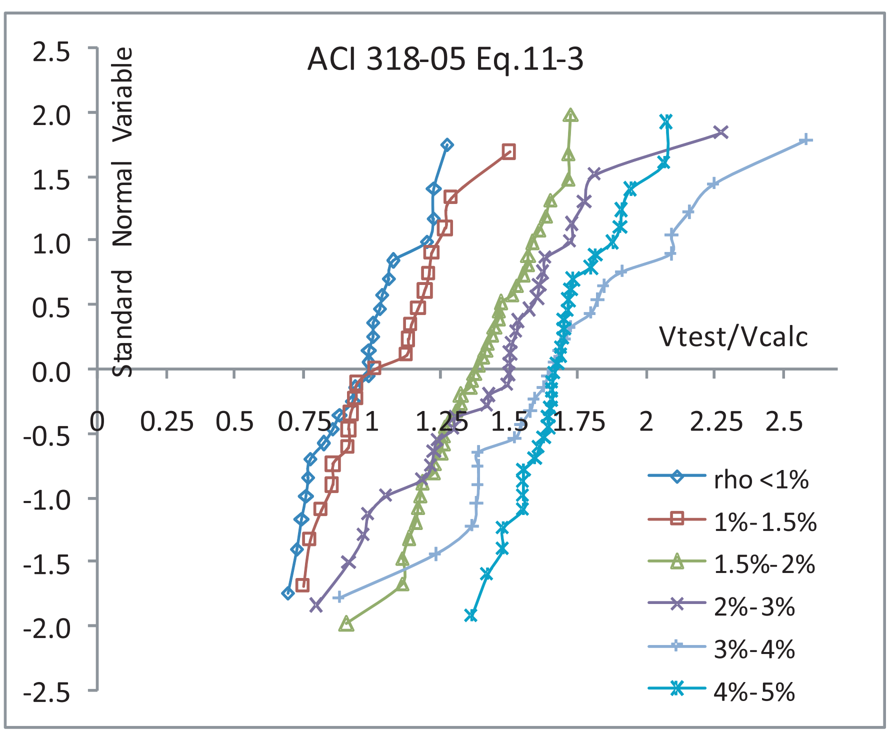

Figure 1.
CDF for Different Longitudinal Reinforcement Ratios ρ [%] – Equation 11-3

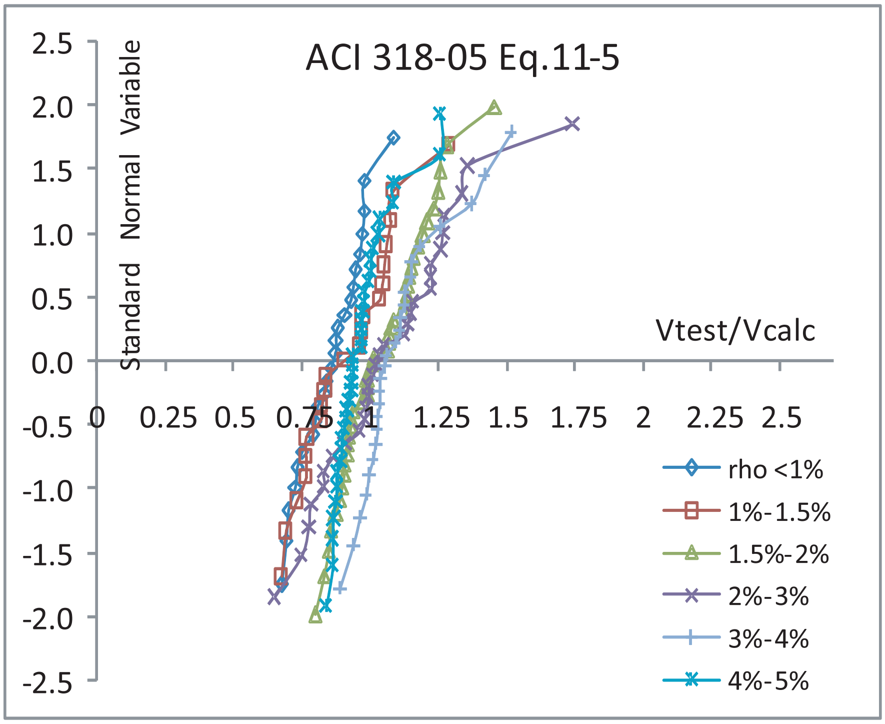

Figure 2.
CDF for Different Longitudinal Reinforcement Ratios ρ [%] – Equation 11-5

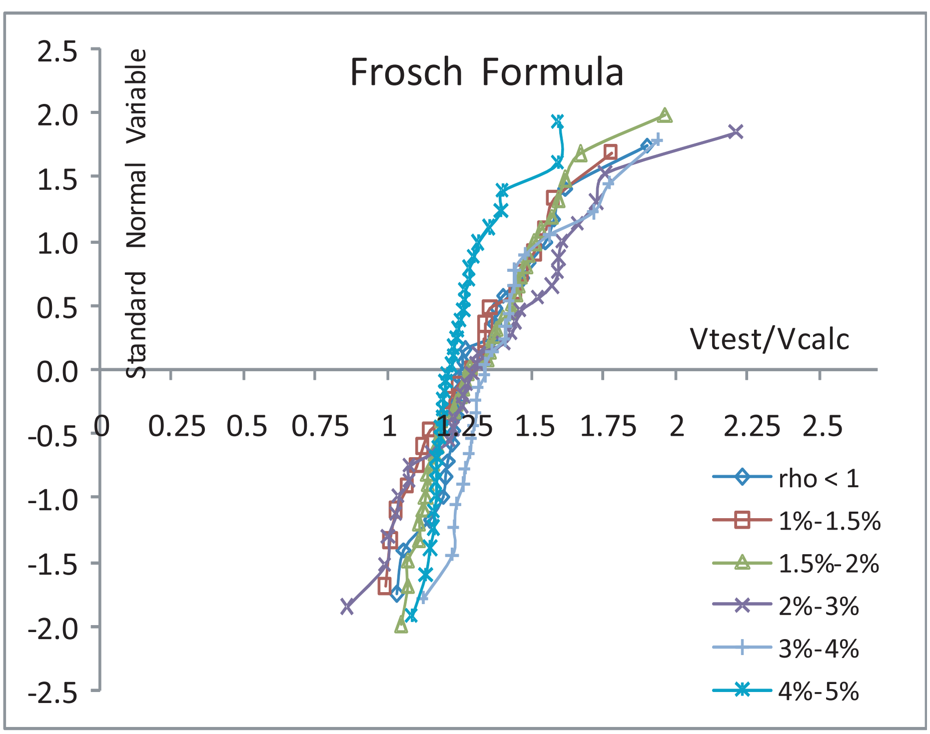

Figure 3.
CDF for Different Longitudinal Reinforcement Ratios ρ [%] – Frosch Formula

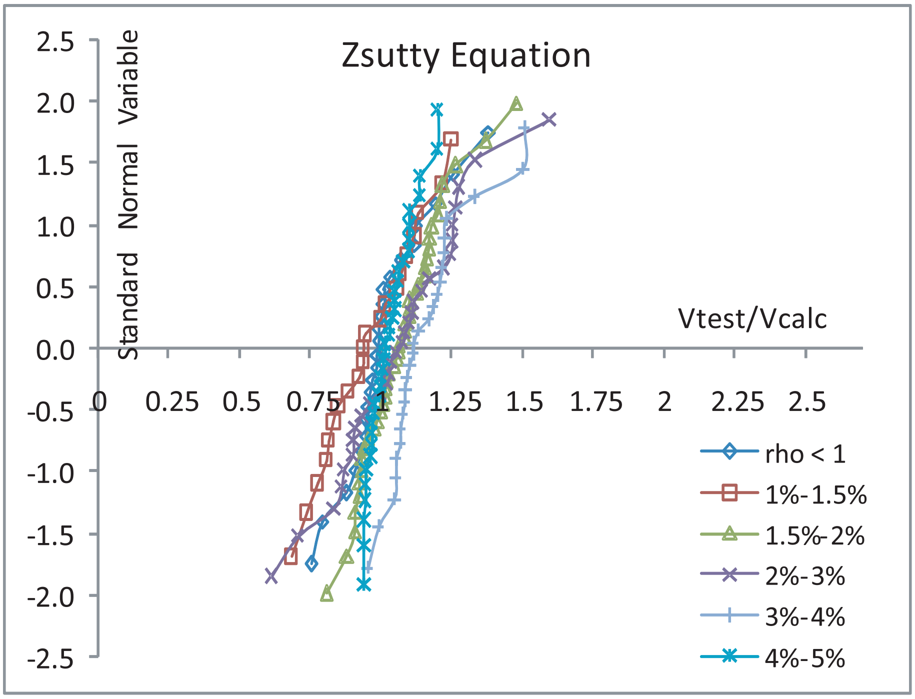

Figure 4.
CDF for Different Longitudinal Reinforcement Ratios ρ [%] – Zsutty Equation

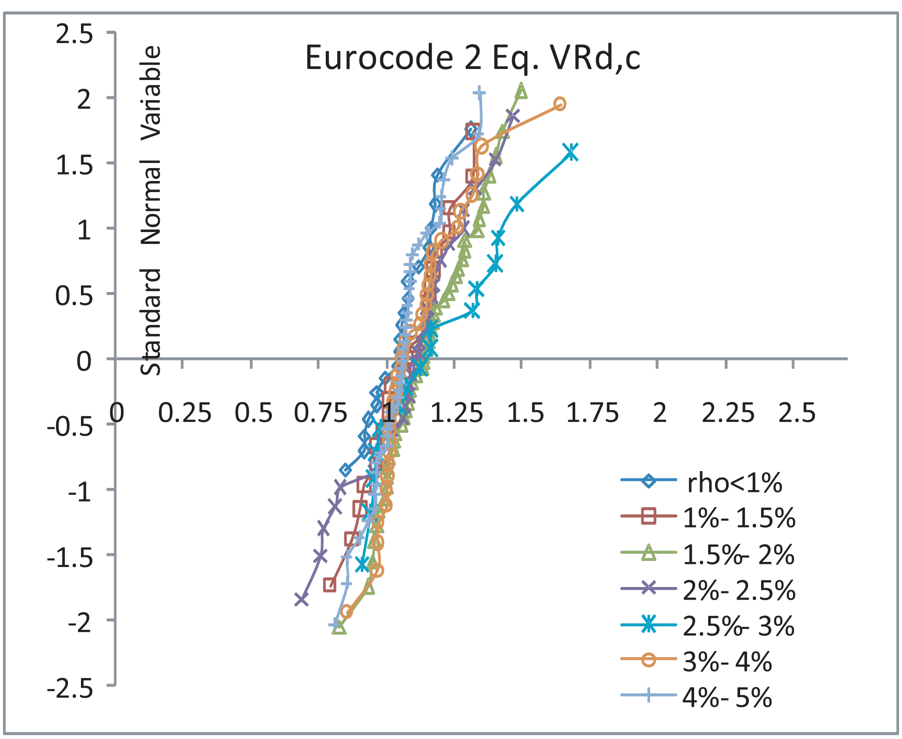

Figure 5.
CDF for Different Longitudinal Reinforcement Ratios ρ [%] – Eurocode 2

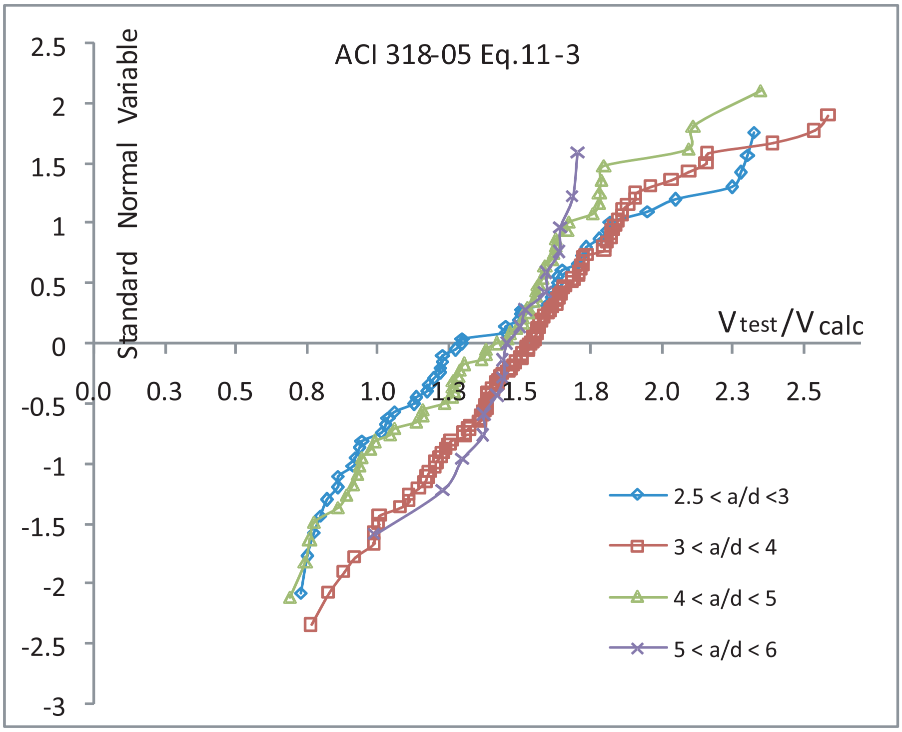

Figure 6.
CDF for Different Shear Span Ratio a/d – Equation 11-3

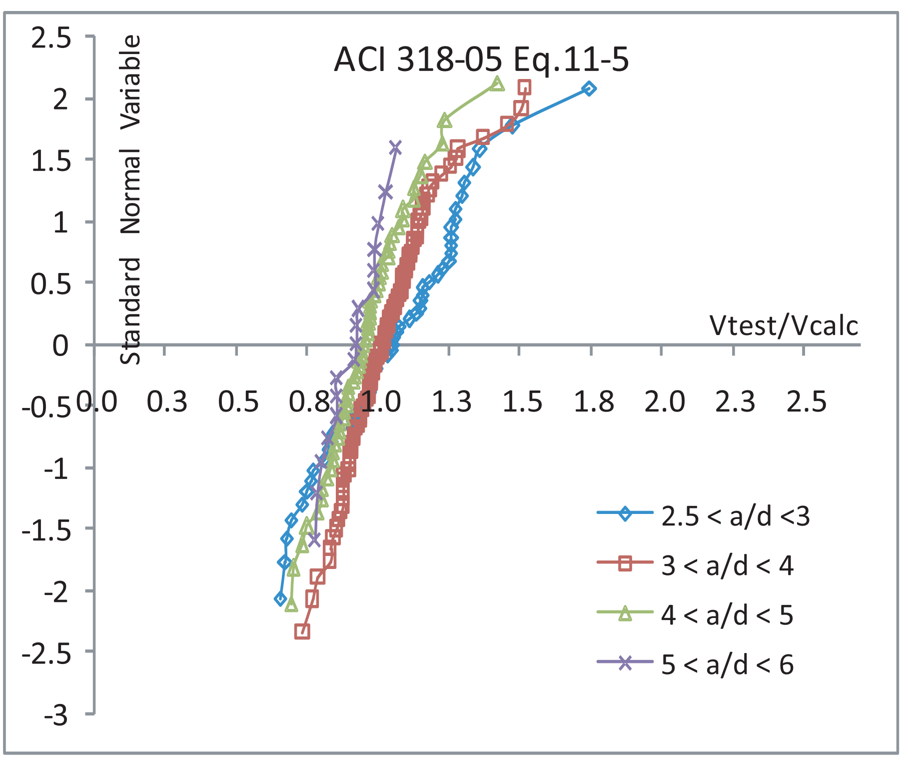

Figure 7.
CDF for Different Shear Span Ratio a/d – Equation 11-5

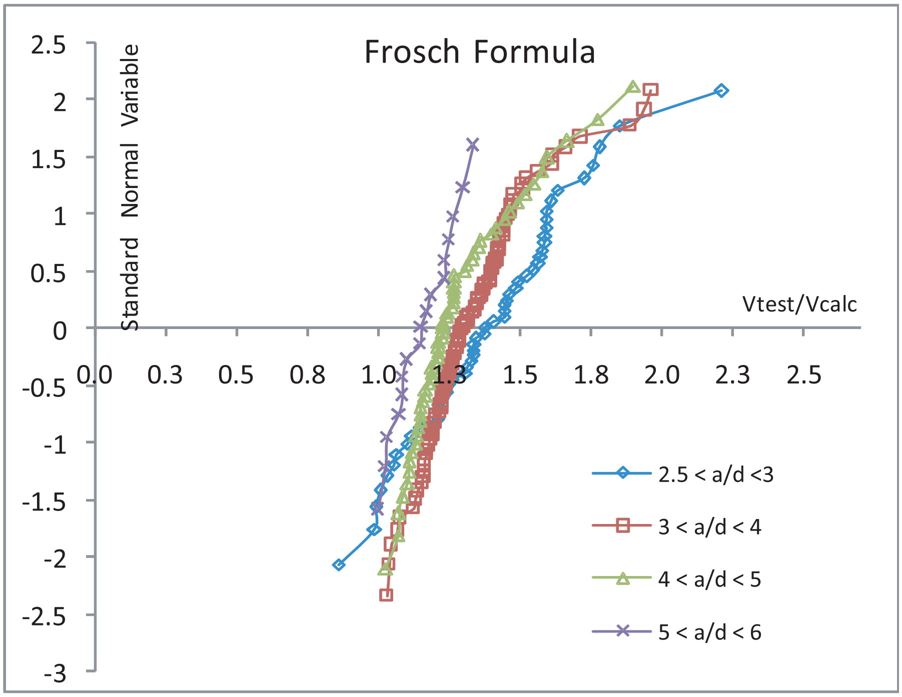

Figure 8.
CDF for Different Shear Span Ratio a/d – Frosch Formula

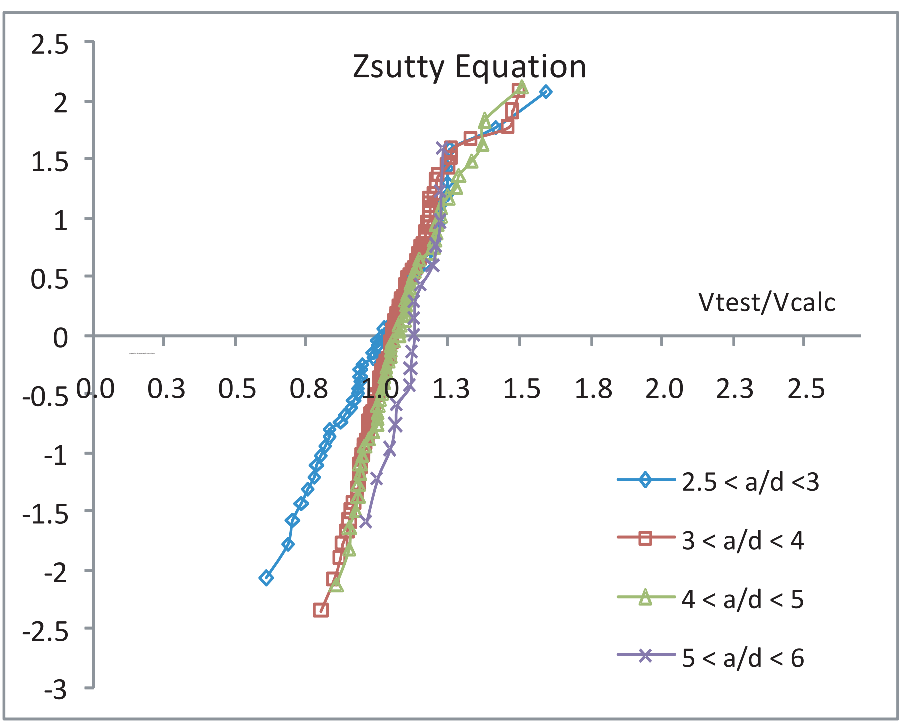

Figure 9.
CDF for Different Shear Span Ratio a/d – Zsutty Equation

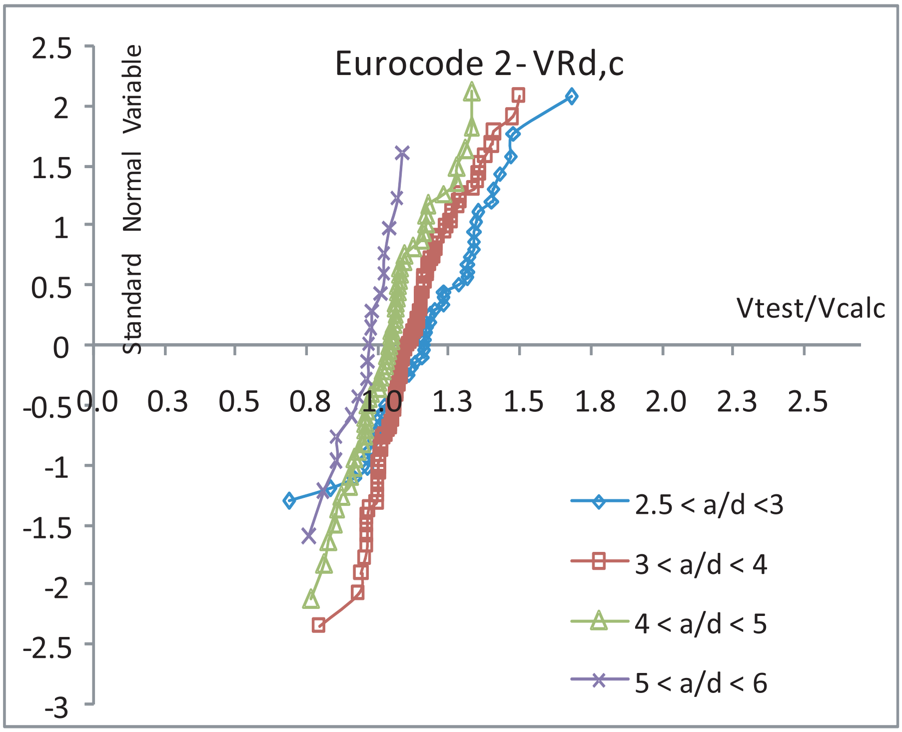

Figure 10.
CDF for Different Shear Span Ratio a/d – Eurocode 2

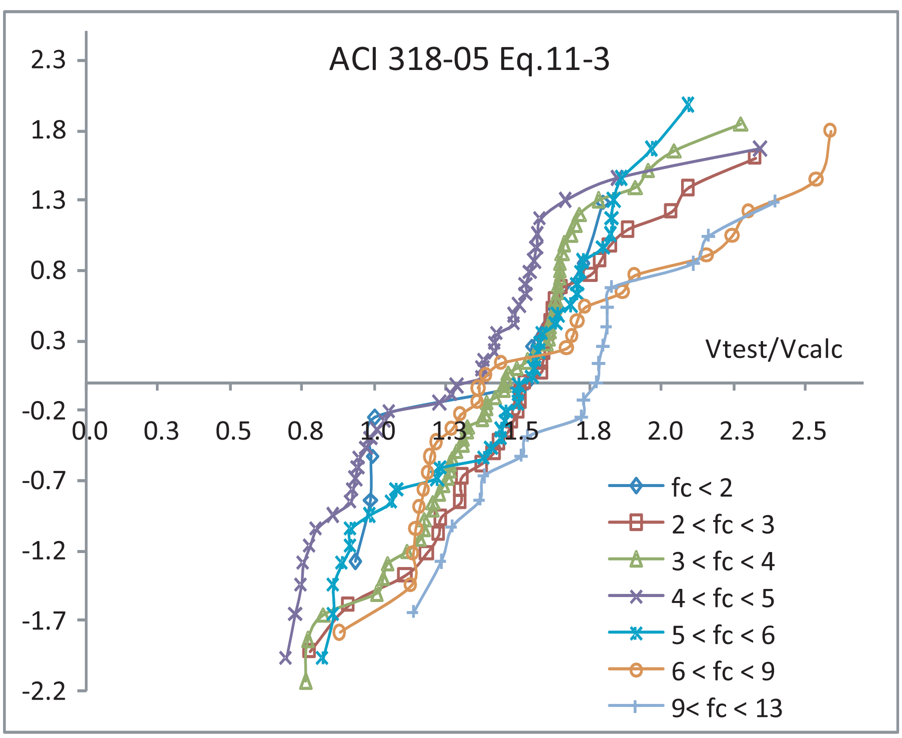

Figure 11.
CDF for Concrete Compressive Strengths – Equation 11-3

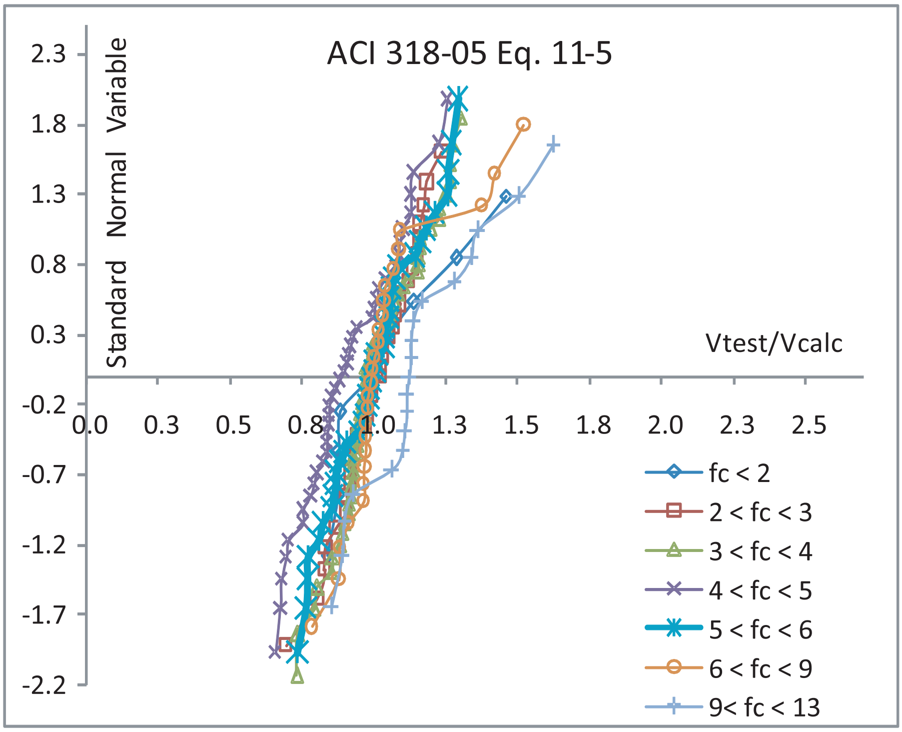

Figure 12.
CDF for Concrete Compressive Strengths – Equation 11-5

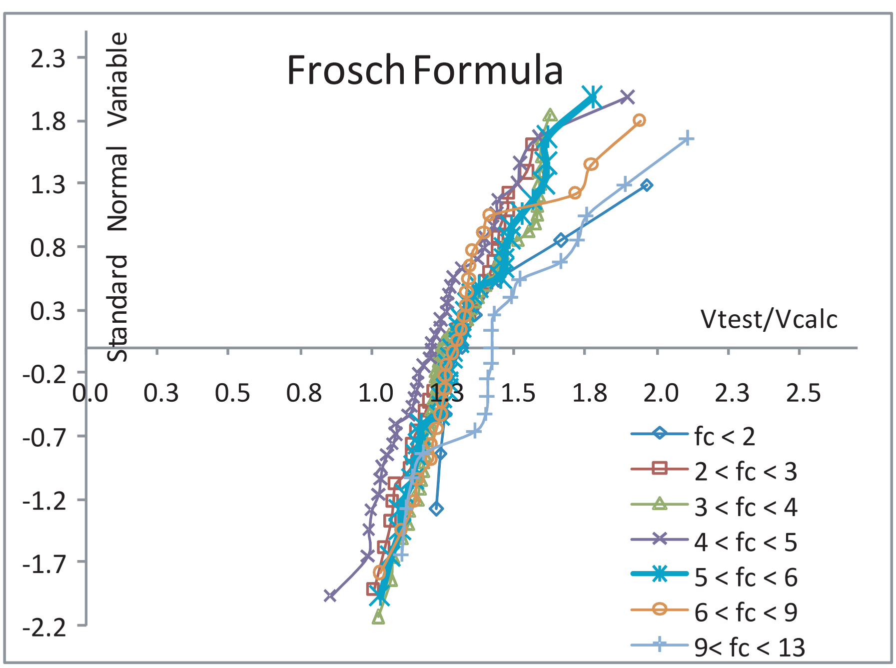

Figure 13.
CDF for Concrete Compressive Strengths – Frosch Formula

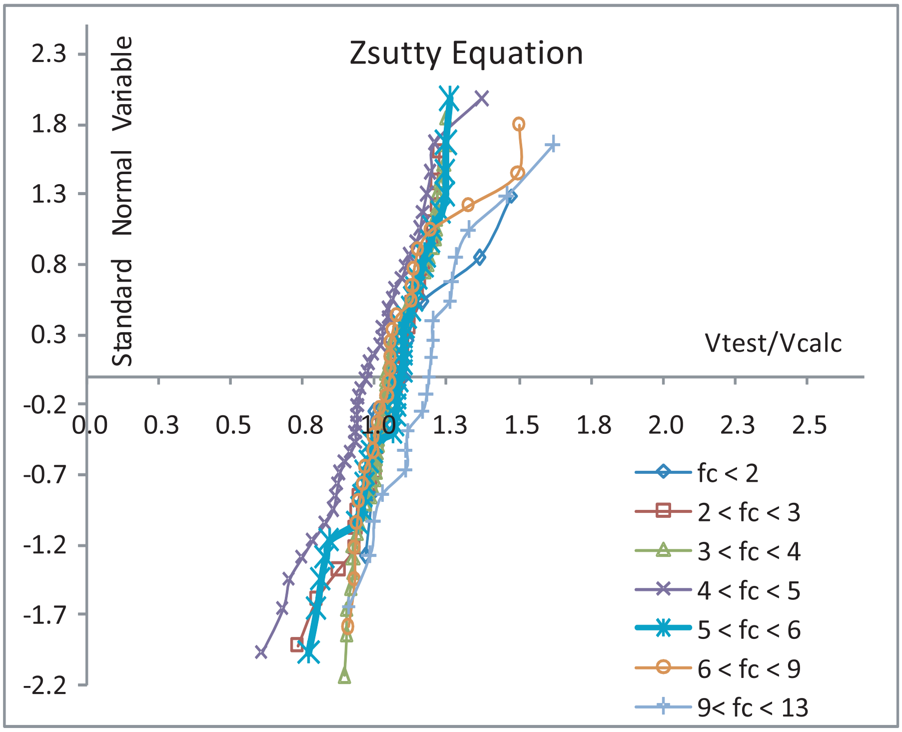

Figure 14.
CDF for Concrete Compressive Strengths – Zsutty Equation

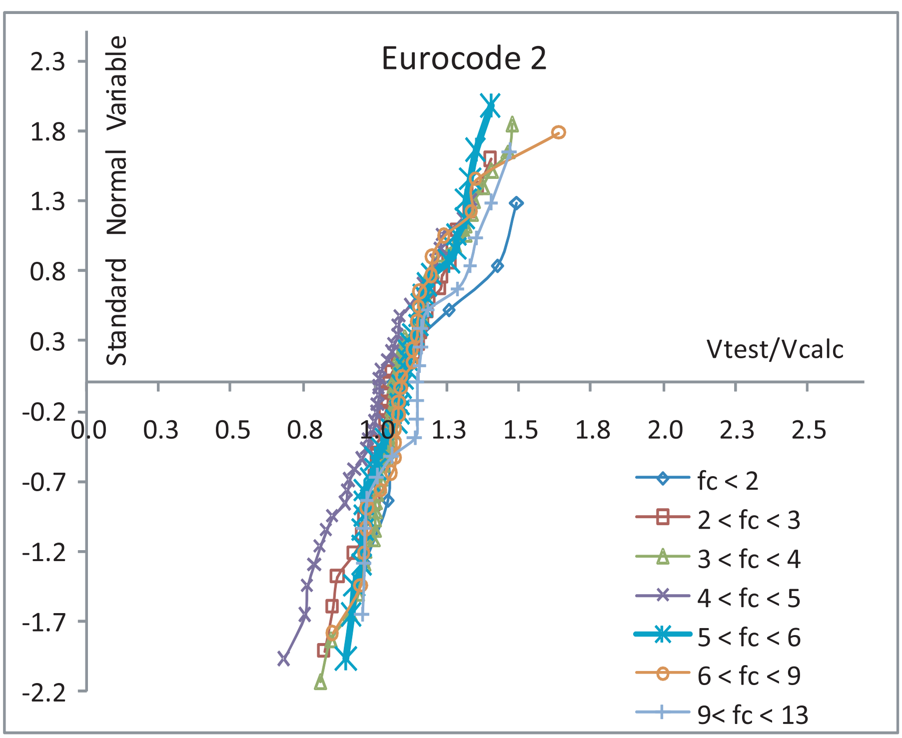

Figure 15.
CDF for Concrete Compressive Strengths – Eurocode 2

Table 5.
Statistical parameters of the bias factor for the considered methods

	ACI Eq.11-3	Frosch Equation	ACI Eq.11-5	Zsutty Equation	Eurocode 2 VRd,c
Mean Value	1.5	1.3	1.0	1.1	1.1
Standard Deviation	0.45	0.21	0.17	0.15	0.16
Coefficient of Variation	0.30	0.16	0.17	0.14	0.14

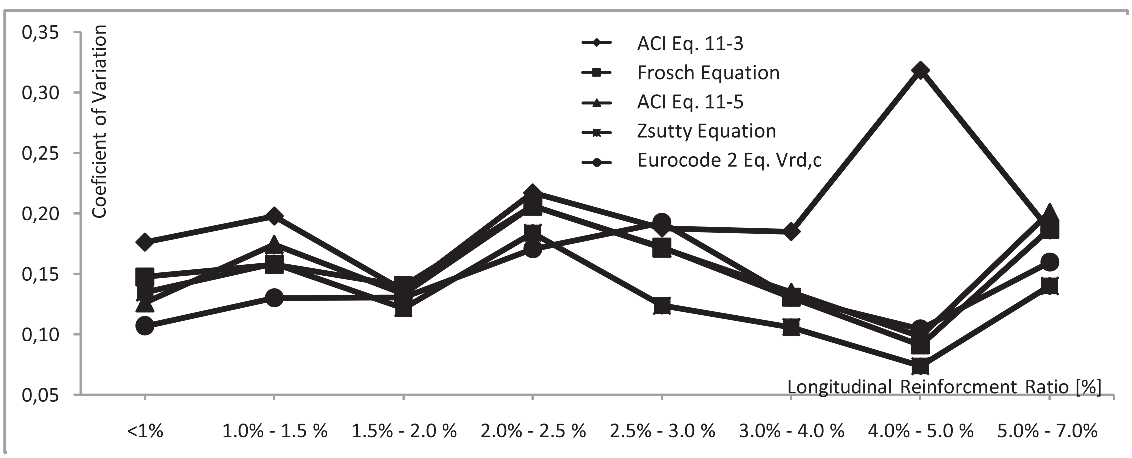

Figure 16.
Coefficient of variation for different methods as a function of ρ [%]

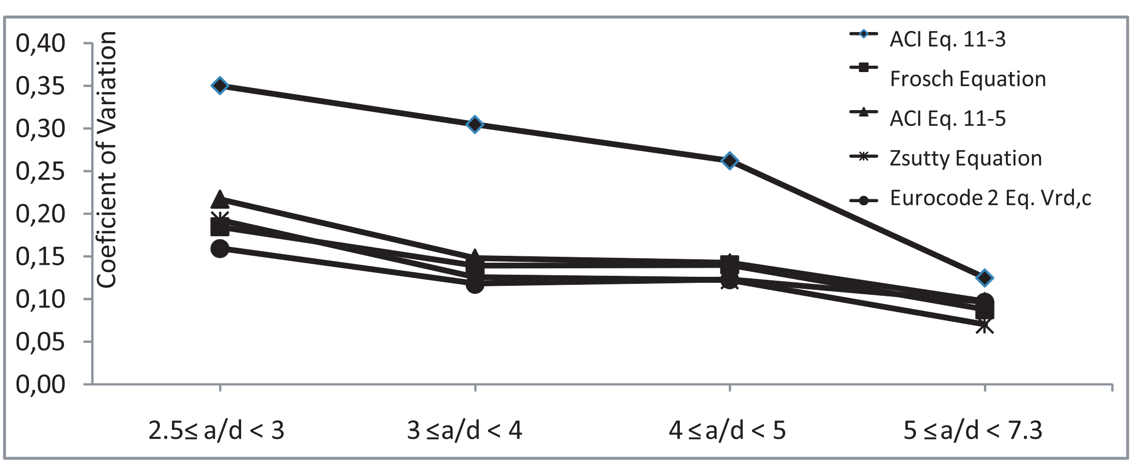

Figure 17.
Coefficient of variation for different methods as a function of a/d ratio

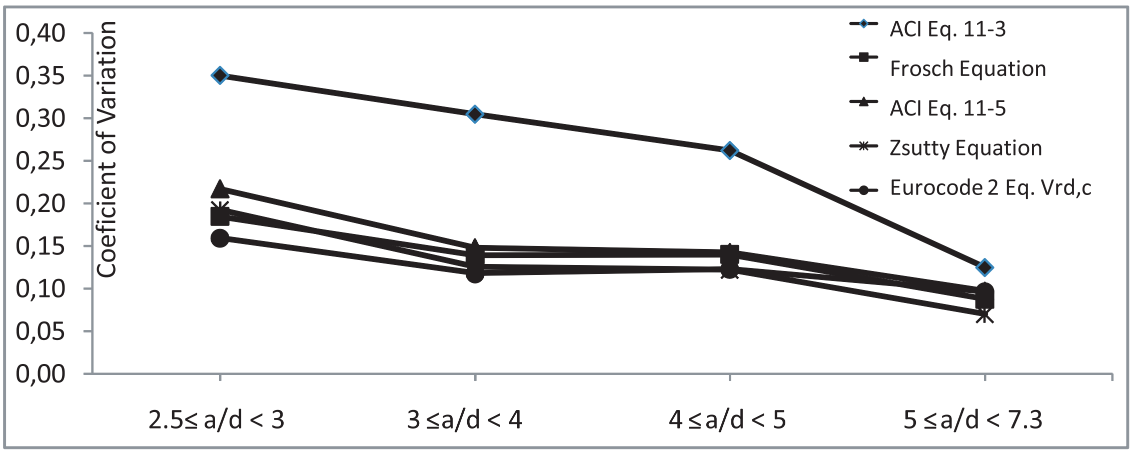

Figure 18.
Coefficient of variation for different methods as a function of f_c'.

ACKNOWLEGMENTS

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REFERENCES

[1] ACI Committee 318, Building Code Requirements for Reinforced Concrete *(ACI 318-95)* and Commentary-ACI318RM-95, American Concrete Institute, Farmington Hills, 1995, p. 436.

[2] *Tureyen A. K., and Frosh R. J.*; Concrete Shear Strength: Another Perspective, ACI Structural Journal, Vol. 100, Nr. 5, September-October 2003, pp. 609-615.

[3] *Zsutty T. C.*; Beam Shear Strength Prediction by analysis of Existing Data. ACI Struct. J., v. 65, Nov., 1968, pp. 943-951.

[4] Eurocode 2 – EN 1992-1-1:2004. Design of Concrete Structures. Part 1: General Rules and Rules for Buildings, CEN, Brussells, December 2004, p. 215.

[5] ACI-ASCE Committee 326 Shear and Diagonal Tension, ACI Journal, Proceedings, V.59, No. 1, 2, and 3, 1962, pp. 1-30, 277-334, 352-396.

[6] *Nowak A. S., Collins K. R.*, Reliability of Structures. McGraw-Hill International Editions, Civil Engineering Series, Singapore, 2000, p. 338.