

ANALYSIS OF A LAP-JOINT IN A THIN-WALLED STRUCTURE UNDER COMBINED BENDING AND SHEARING LOAD

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Abstract

Basing on an experimentally verified mathematical nonlinear model describing the static behavior of an arbitrary joint of thin-walled profiles a numerical analysis of a five-blind-bolt lap-joint has been carried out. Diagrams of three cases of limit curves: I – limit load bearing capacity, II – limit displacements and III – limit of the mathematical model have been plotted. Additionally, two limit surfaces of the joint have been presented. The main aim of the contribution is an analysis of the mutual influence of the shearing force and the moment on the corresponding rigidities. To show this, the interaction relations between rotation, transverse and axial rigidities have been expressed in terms of rigidity reduction coefficients. Their diagrams are presented in form of contour plots. The analysis has been carried out for different angles of the direction of the shearing force. Contour plots of displacements and the corresponding forces are shown for selected bolts, too. It has been shown that the coefficients of rigidity reduction are not sensitive to changes of the angle of transverse forces direction, whereas some forces in the bolts change considerably with this angle.

Streszczenie

Bazując na doświadczalnie zweryfikowanym nieliniowym modelu opisującym statyczne zachowanie się dowolnego połączenia cienkościennych profili przeprowadzono numeryczną analizę połączenia zakładkowego na pięć łączników. Zostały narysowane wykresy trzech przypadków krzywych granicznych: I – granicy nośności, II – granicy przemieszczeń i III – granicy modelu matematycznego. Głównym celem pracy jest analiza wzajemnego wpływu siły ścinającej i momentu na odpowiadające sztywności. Aby to pokazać, relacje interakcyjne pomiędzy obrotem, a sztywnościami poprzecznymi i osiowymi zostały wyrażone poprzez współczynniki redukcji sztywności. Zostały one pokazane w postaci map konturowych. Te analizy zostały przeprowadzone dla różnych kątów kierunku siły ścinającej. Dla wybranych łączników przedstawiono też mapy konturowe przemieszczeń i odpowiadających im sił. Pokazano, że współczynniki redukcji sztywności nie są wrażliwe na zmiany kąta kierunkowego sił poprzecznych, natomiast niektóre siły w łącznikach zmieniają się istotnie wraz z tym kątem.

Keywords: Nonlinear; Lap-joint; Limit state; Thin-walled; Interaction; Blind bolt; Lightweight; Rigidity reduction.

1. INTRODUCTION

For several recent years a crucial demand for thin-walled structures made of cold-formed profiles has been observed. Many overseas companies erected in Poland hundreds of buildings according to their structural system solutions; this was described in Ref. [1]. The rich assortment of thin-walled cross-sections offered by Polish metallurgic factories can encourage

development of an own structural system with joints enabling a reliable and quick assembly on the construction site. These requirements are satisfied by blind bolts BOM R16-6, diameter \varnothing 13.6 mm [2]. They permit to join walls of a thickness $3.0 \div 6.0$ mm with open or semi-open economic closed cross-section profiles.

The aim of the investigations is an attempt to recog-

nize behavior of blind-bolt joint in thin-walled structures. A mathematical model of the joint has been proposed and experimentally verified. General governing equations of an arbitrary multi-bolt joint loaded with a moment and shearing force has been formulated in Refs [3, 4, 5, 6]. Constitutive relations in the form of exponential function have been evaluated by a function of nonlinear regression, according to regulations quoted in [7].

Mathematical model has been implemented within the system *Mathematica* [8], by Walentyński R. and verified experimentally, [4], by Wuwer W. A good coincidence between numerical and experimental data has been observed in [9].

According to the proposed model, which has been verified experimentally, we can estimate the load capacity and rigidity of an arbitrary joint, and to develop limit curves and limit surfaces of displacements and the load capacity for an arbitrary system of loads. Previously we should evaluate the parameters of a constitutive relation. This can be done by an experiment developed according to [7].

Contemporary computer systems for numerical analyses based on the Finite Element Method, for example Robot Millennium, permit also nonlinear releases of bars. They enable us to compute bar structures with flexible nodes. The values of three rigidities in the joint, depending on rotation and two orthogonal displacements occurring between connected walls of cold-formed profiles have a crucial influence on the internal forces and vertical and horizontal deflection, as shown in [9]. The function in the computer system does not take into account the interaction between three instantaneous rigidities. They allow only to define, for example, the rotation rigidity as a function of the moment. The shearing force has been neglected.

We have developed a proposal of evaluating the rigidities of flexible nodes. The rigidities depend on three parameters of degradation ω_M , ω_H and ω_V . These parameters take only into account variations of the global plastic destruction of the corresponding force. For example, ω_M is a function of momentum. The introduced coefficients v_M , v_H and v_V take into account the mutual influence of forces on the rigidities. For example, v_M depends on the moment and the components of the shearing force.

The paper is illustrated by contour and parametric plots of some displacement, the corresponding forces in blind bolts, the coefficients v_M , v_H and v_V , the limit curves and limit surfaces.

This paper is a further development of the results presented in [13].

2. PROBLEM DESCRIPTION

2.1. Physical model

We will deal in this paper with a joint that is presented in Fig. 1. The 5-blind-bolt joint is loaded with the moment M and the transverse W sloped at the angle α_W to the vertical axis y . Due to the symmetry of the joint the practical analysis can be reduced to the angle $\alpha_W \in \langle 0^\circ, 45^\circ \rangle$.

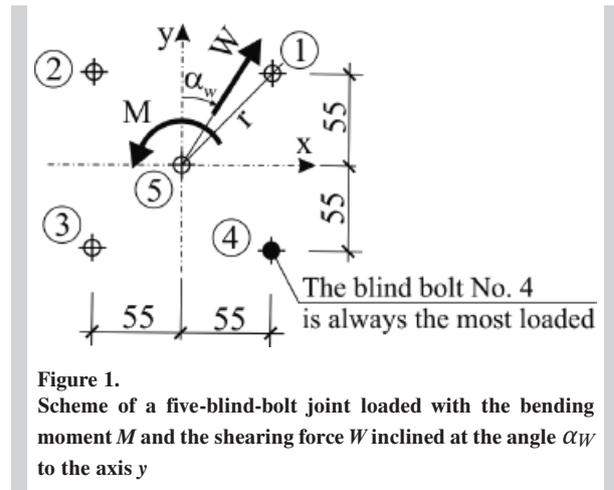


Figure 1. Scheme of a five-blind-bolt joint loaded with the bending moment M and the shearing force W inclined at the angle α_W to the axis y

It can be proved that in this case the most saddled bolt is the fourth one, Fig. 1. Figure 2 illustrates an analysis for 3 angles $\alpha_W \in \langle 0^\circ, 90^\circ \rangle$ showing that this bolt always takes over most of the load.

It is also easily visible that the fifth bolt does not take part in the bearing of the moment.

2.2. System of equations for a 5-blind-bolt joint

A general system of governing equations is presented in the dissertation [4]. For the considered problem the system of equations has been expanded, and we have the following equations:

– 3 equations of equilibrium

$$V - v \left(\frac{S_1}{\delta_1} + \frac{S_2}{\delta_2} + \frac{S_3}{\delta_3} + \frac{S_4}{\delta_4} + \frac{S_5}{\delta_5} \right) - \frac{r \cdot \phi}{\sqrt{2}} \left(\frac{S_1}{\delta_1} - \frac{S_2}{\delta_2} - \frac{S_3}{\delta_3} + \frac{S_4}{\delta_4} \right) \equiv 0 \quad (1)$$

$$H - u \left(\frac{S_1}{\delta_1} + \frac{S_2}{\delta_2} + \frac{S_3}{\delta_3} + \frac{S_4}{\delta_4} + \frac{S_5}{\delta_5} \right) - \frac{r \cdot \phi}{\sqrt{2}} \left(\frac{S_1}{\delta_1} + \frac{S_2}{\delta_2} - \frac{S_3}{\delta_3} - \frac{S_4}{\delta_4} \right) \equiv 0 \quad (2)$$

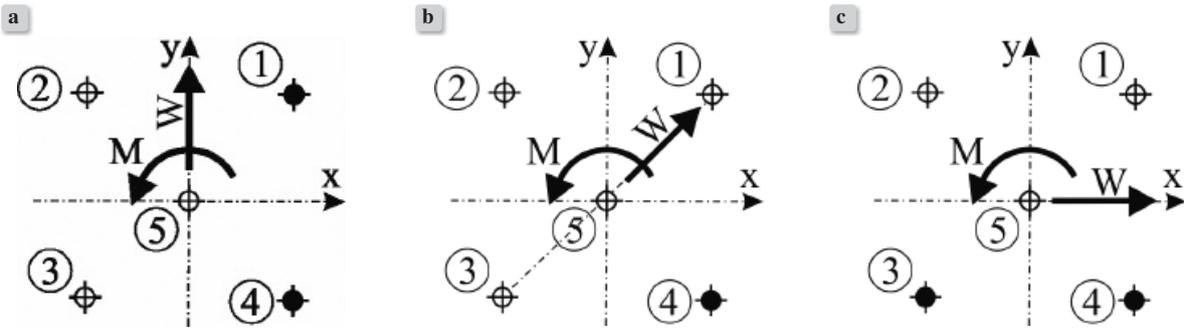


Figure 2. Extreme cases of loading of a five-blind-bolt joint with the forces M and W ; a) $\alpha_W = 0$, b) $\alpha_W = \pi/4$, c) $\alpha_W = \pi/2$

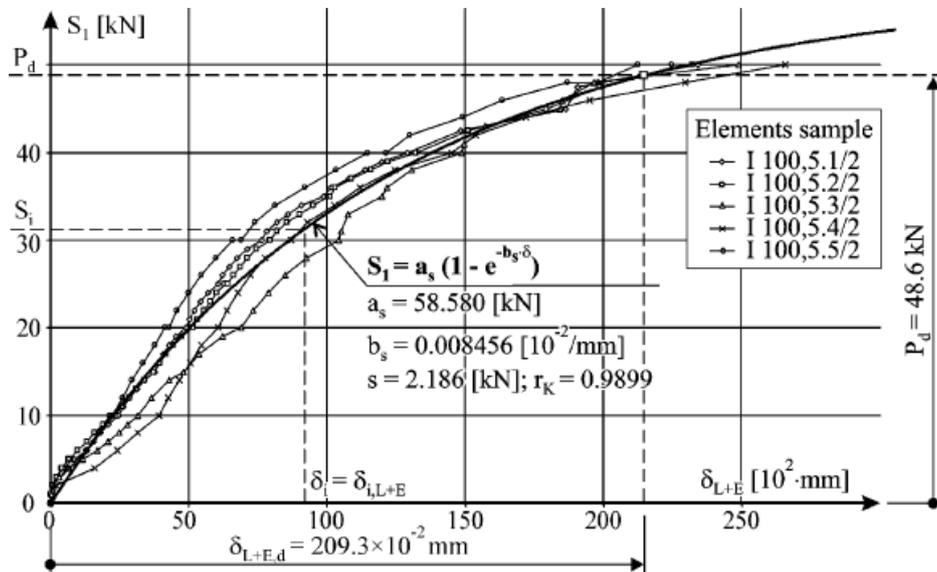


Figure 3. Relation $S_i - \delta_{L+E}$ when walls 5.0 mm thick are joined in five test elements “I”

$$\sqrt{2} \frac{M}{r} + u \left(\frac{S_1}{\delta_1} + \frac{S_2}{\delta_2} - \frac{S_3}{\delta_3} - \frac{S_4}{\delta_4} \right) + \dots \quad (3)$$

$$- v \left(\frac{S_1}{\delta_1} - \frac{S_2}{\delta_2} - \frac{S_3}{\delta_3} + \frac{S_4}{\delta_4} \right) - \sqrt{2} \cdot r \cdot \phi \left(\frac{S_1}{\delta_1} + \frac{S_2}{\delta_2} + \frac{S_3}{\delta_3} + \frac{S_4}{\delta_4} \right) \equiv 0$$

– 5 geometrical equations

$$\delta_1 - \sqrt{\left(u - \frac{r \cdot \phi}{\sqrt{2}}\right)^2 + \left(v + \frac{r \cdot \phi}{\sqrt{2}}\right)^2} \equiv 0 \quad (4)$$

$$\delta_2 - \sqrt{\left(u - \frac{r \cdot \phi}{\sqrt{2}}\right)^2 + \left(v - \frac{r \cdot \phi}{\sqrt{2}}\right)^2} \equiv 0 \quad (5)$$

$$\delta_3 - \sqrt{\left(u + \frac{r \cdot \phi}{\sqrt{2}}\right)^2 + \left(v - \frac{r \cdot \phi}{\sqrt{2}}\right)^2} \equiv 0 \quad (6)$$

$$\delta_4 - \sqrt{\left(u + \frac{r \cdot \phi}{\sqrt{2}}\right)^2 + \left(v + \frac{r \cdot \phi}{\sqrt{2}}\right)^2} \equiv 0 \quad (7)$$

$$\delta_5 - \sqrt{u^2 + v^2} \equiv 0 \quad (8)$$

– constitutive equation 1

$$S_i \equiv a_s (1 - e^{-b_s \cdot \delta_i}) \quad (9)$$

where:

S_i – forces in blind bolts,

δ_i – mutual displacement of joined profile walls in the direction of the forces S_i ,

u – mutual horizontal displacement of the walls in the joint,

- v – mutual vertical displacement of the walls in the joint,
- ϕ – mutual rotation displacement of the walls in the joint,
- r – radius, see Fig. 1,
- H – horizontal component of the shearing force
- V – vertical component of the shearing force
- M – moment, see Fig. 1.

The parameters a_s and b_s in the constitutive relation, Eqn 9, have been evaluated by nonlinear regression

from experimental data, see Fig. 3.

2.3. Symbolic computation

The system of equation (1-9) can be reduced to a system of three equations with unknown values of the displacements u , v and ϕ . This was done by applying the computer system *Mathematica*. Due to the complicated form of the equations the system is not presented in this paper.

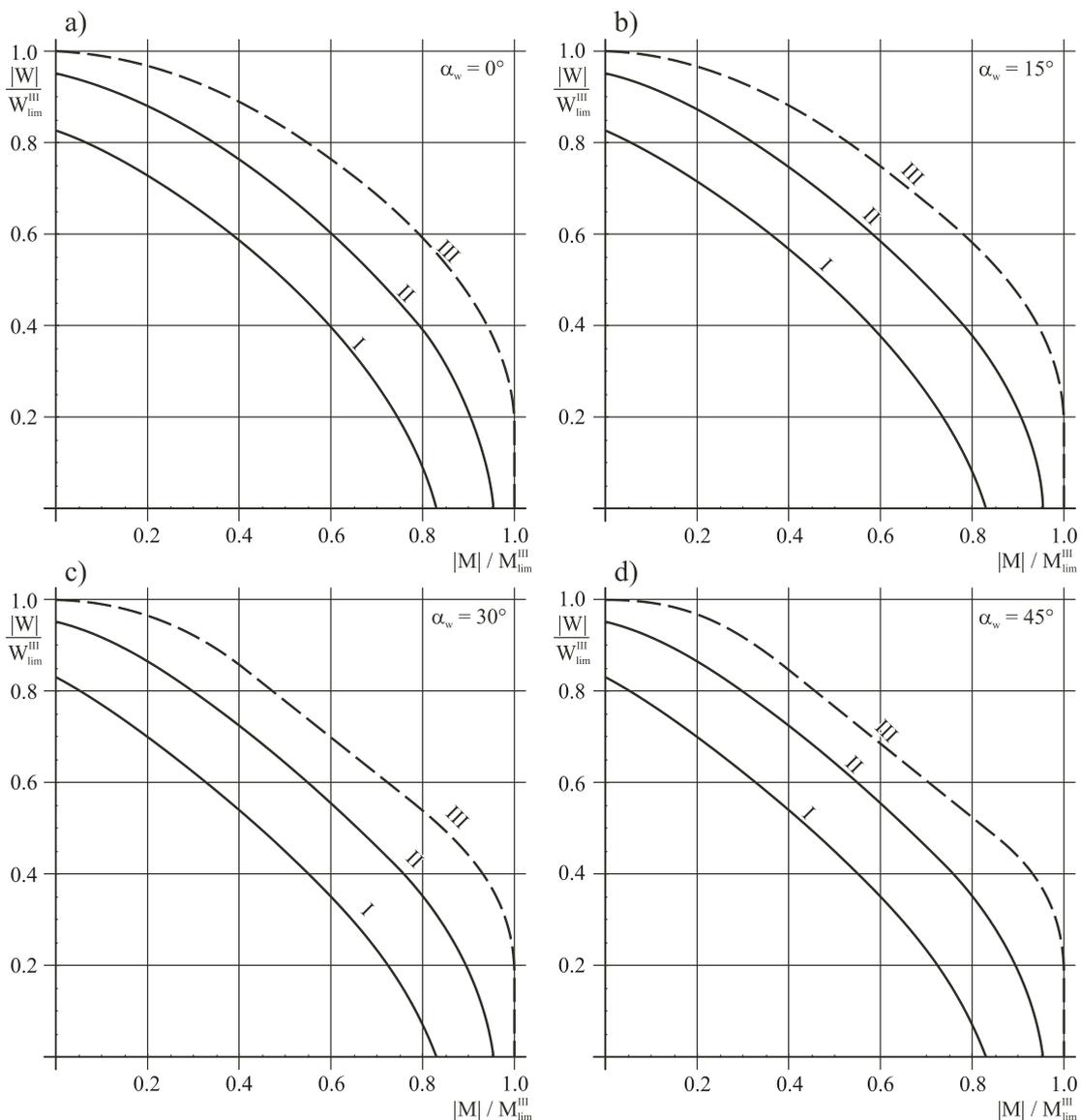


Figure 4. Boundary curves I, II and III of a five-blind-bolt joint loaded with the forces M and W , related to the third case of the boundary state concerning four cases of loading: a) when $\alpha_W = 0^\circ$, b) when $\alpha_W = 15^\circ$, c) when $\alpha_W = 30^\circ$, d) when $\alpha_W = 45^\circ$

3. NUMERICAL ANALYSIS OF A LAP-JOINT

The obtained system of the free nonlinear algebraic equation has been solved for the given data of forces. The algorithm was implemented within the *Mathematica* system.

3.1. Limit states

The solution is limited with regard to the domain of the shearing force W and moment M . These limitations have a different origin. The first limitation is due to the fact that the maximum force in any blind bolt of the joint cannot exceed the design value. Statistically it was in the considered case estimated as:

$$\max(S_i) \leq S_{lim}^I = P_d = 48.6 \text{ kN} \quad (10)$$

This value of force implies that the mutual displacement of the profile walls $\delta_{L+E,d} = 209.3 \cdot 10^{-2} \text{ mm}$, see Fig. 1. The first limit state can be regarded as an engineering one.

The second limit state comes from the limitation quoted in [7] concerning the value of the characteristic load-bearing capacity of the blind bolt. It has been stated that values of the obtained forces cannot be taken into account when the mutual displacement of the joint exceeds: $\max(\delta_i) \leq \delta_{gr} \equiv 3.0 \text{ mm}$; that implies that the characteristic value of the bolt load capacity is

$$\max(S_i) \leq P_{m,av} = S_{lim}^{II} = 55.85 \text{ kN} \quad (11)$$

This value was used as a basis for the estimation of the first limit state in the statistical way and the second limit state of the blind-bolt joint. This limit state can be regarded as a physical one.

The mathematical model presented in point 2.2 is limited by the domain connected with the fact that the constitutive equation (9) has a horizontal asymptote. It implies that the force in any bolt may not cross the limit:

$$\max(S_i) \leq S_{lim}^{III} \equiv P_{destr} \equiv a_s \equiv 58.58 \text{ kN} \quad (12)$$

This limit state can be regarded as a mathematical one.

In the analysis of the problem we can distinguish two special cases. The first one when the joint is loaded with the shearing force only, $M \equiv 0$. In that case the shearing force is taken to be uniform in all five bolts, so the limit value can be evaluated exactly:

$$W^X = 5 \cdot S_{lim}^X \quad (13)$$

The second special case is observed when the joint is loaded only with the moment, $W \equiv 0$. The moment concerns only bolts 1-4, and its limit state value can be assessed exactly

$$M^X = 4 \cdot r \cdot S_{lim}^X \quad (14)$$

Symbol X in eqs (13) and (14) stands for I, II and III, depending on the limit state.

According to these statements the limit curves have been evaluated for 4 different angles of the direction of the force W. Figure 4 presents 3 limit curves. The lowest light-grey line refers to the first limit state I, Eqn 10. The second line dark-grey line refers to the second limit state, Eqn 11. The highest curve (black) refers to the third limit state, Eqn 12. The domain of each diagram is normalized to the dimensionless area $\langle 0,1 \rangle \times \langle 0,1 \rangle$.

The variation of the curves is small with regard to the angle. Nevertheless, it can be noticed that the limit curves in Fig. 4a embrace the largest areas, whereas in Fig. 4d the smallest ones. This leads to the conclusion that it is more favourable if the direction of the shearing force coincides with the direction of the axes x or y (see Fig. 2a or 2c), than if it has a sloping direction (see Fig. 2b). It should be taken into account while designing the connections, as far as possible of course.

Figure 5 presents two limit surfaces of the considered 5-blind-bolt joint. The inner surface represents the first limit state, the outer one represents the third limit state. The surfaces have three planes of symmetry but they are not surfaces of revolution (cf. Figs 4a-d), which shows meridians of these surfaces.

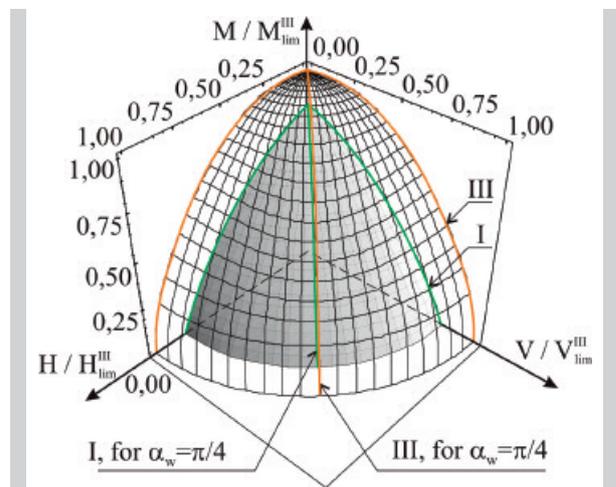


Figure 5. Boundary surfaces of the calculated load-carrying capacity I and of the destructive load-carrying capacity III of a five-blind-bolt joint

3.2. Solution of the problem

The system of equation has been solved for different values of the angle α_W , Fig. 1. This angle changes the direction of the shearing force from vertical, Fig 2a to sloping, Fig 2b. Figures 6 and 7 show diagrams of the mutual displacement and forces in the bolt No. 1.

Comparing these diagrams we see that variations of the angle do not influence these functions much.

The situation is different in case of bolt No. 2 (see Fig. 1). Figures 8 and 9 show that the mutual displacement and forces in the bolt No. 2 are highly sensitive to variations of the angle α_W . The black colour in the upper right-hand corner represents the area outside the mathematical domain of the model. This remark refers also to further diagrams in this paper.

Performing the analysis according to the proposed model we can design properly further experiments, predict the behaviour of the joint, estimate the destruction load and calibrate the load path.

4. RIGIDITY OF THE JOINT

The solution of the system permits to estimate the rigidity of the joint, as has been mentioned in the introduction to the computer analysis of structures with flexible nodes. The rigidity of the joint is according to the proposed nonlinear model a function of all load components [4]. Usually built-in procedures in finite element systems, for example RoboBAT products, allow to subordinate rigidities only to the corresponding forces, for example rotation rigidity is a function of the momentum only. We can show that our model enables us to take into account three forces. It is done by introducing reduction coefficients of rigidity.

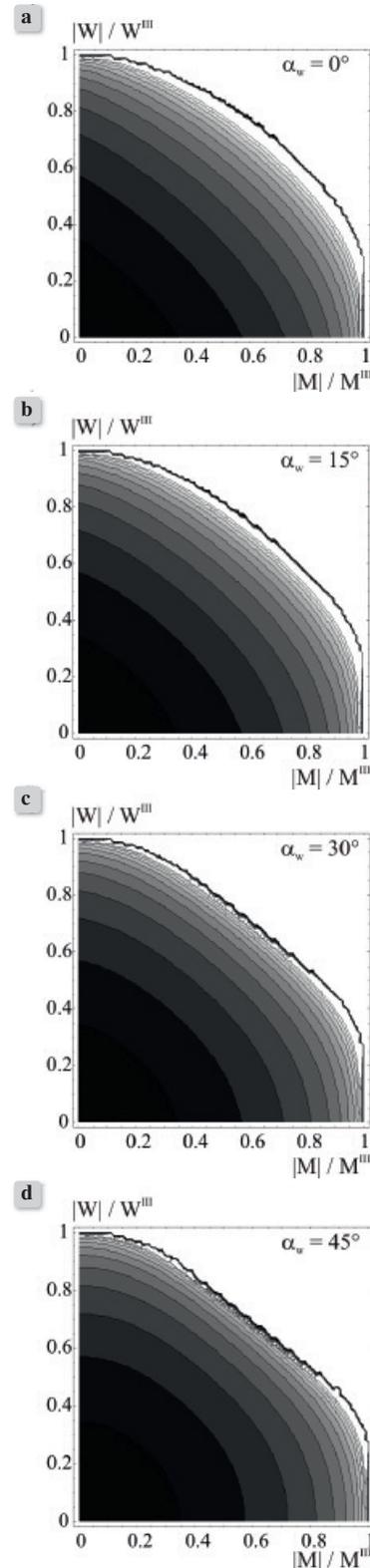


Figure 6. Contour maps of displacements δ_1 occurring between the walls of the bind bolt No. 1: a) when $\alpha_W = 0^\circ$, b) when $\alpha_W = 15^\circ$, c) when $\alpha_W = 30^\circ$, d) when $\alpha_W = 45^\circ$

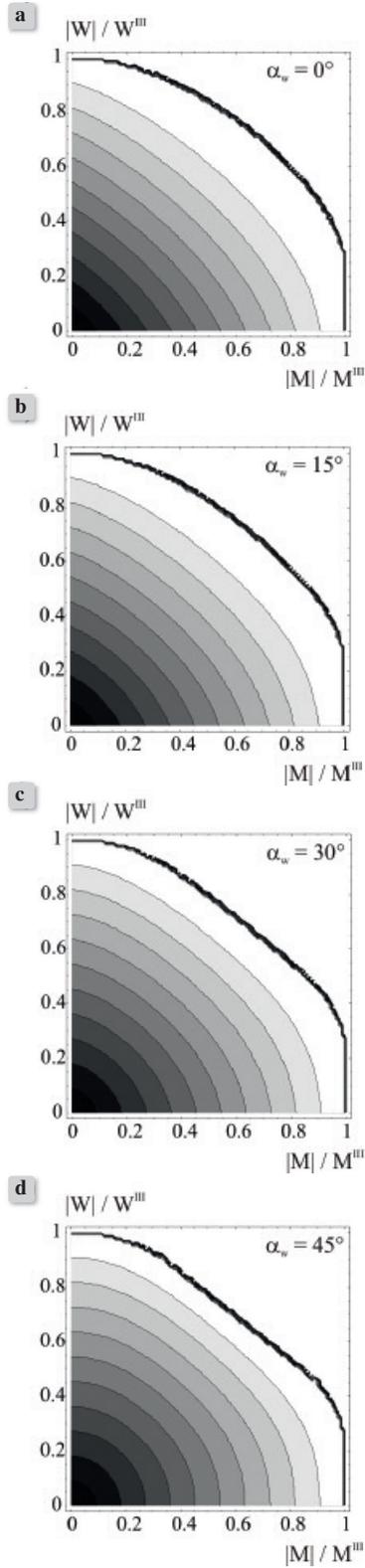


Figure 7. Contour maps the forces S1 affecting respectively the blind bolt No. 1: a) when $\alpha_w = 0^\circ$, b) when $\alpha_w = 15^\circ$, c) when $\alpha_w = 30^\circ$, d) when $\alpha_w = 45^\circ$

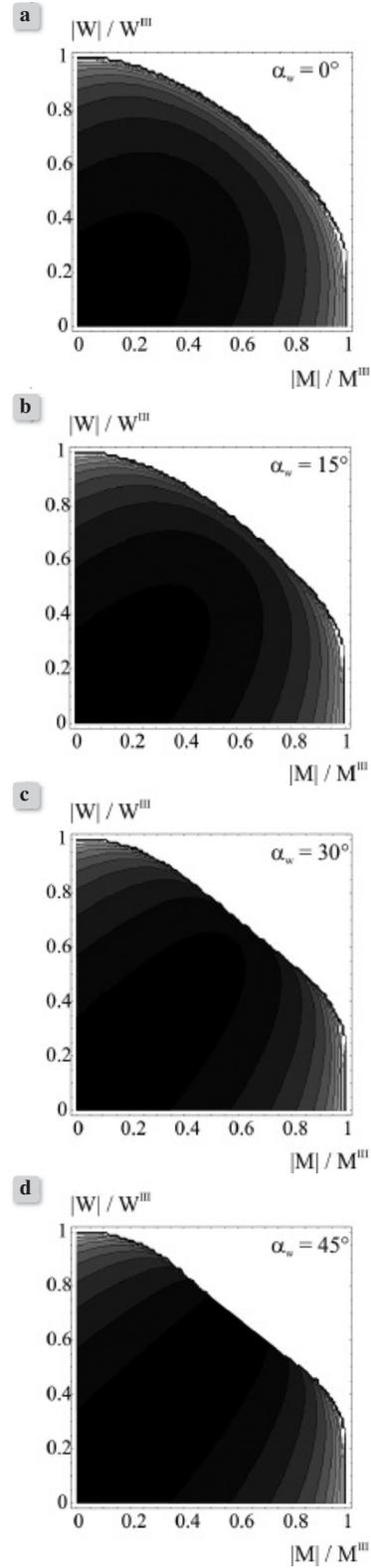


Figure 8. Contour maps of displacements 2 occurring between the walls of the blind bolt No. 2: a) when $\alpha_w = 0^\circ$, b) when $\alpha_w = 15^\circ$, c) when $\alpha_w = 30^\circ$, d) when $\alpha_w = 45^\circ$

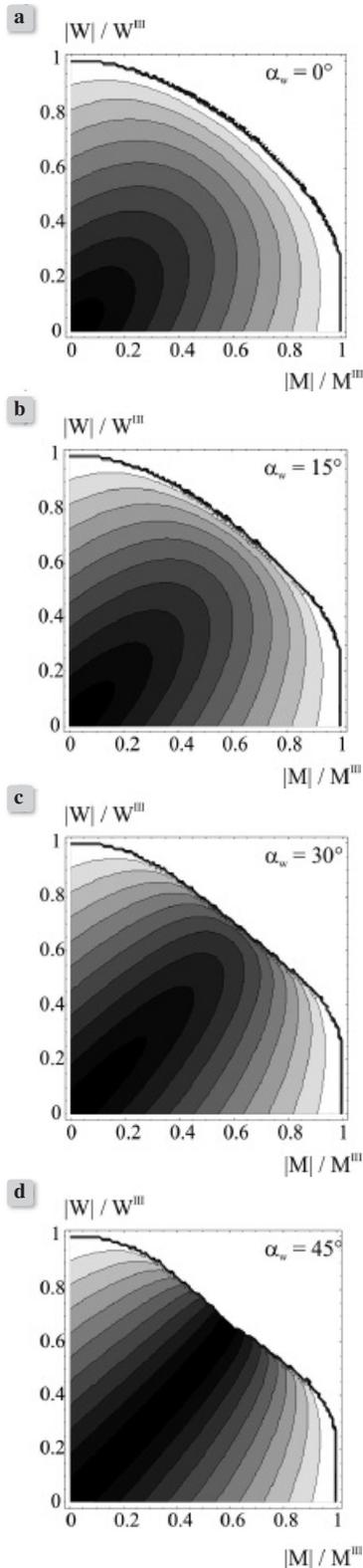


Figure 9. Contour maps of forces S2 affecting respectively the blind bolt No. 2: a) when $\alpha_W = 0^\circ$, b) when $\alpha_W = 15^\circ$, c) when $\alpha_W = 30^\circ$, d) when $\alpha_W = 45^\circ$

We distinguish three instantaneous rigidities of the joint. In the considered case of a 5-blind-bolt joint we have:

– instantaneous rigidity of vertical displacement

$$K_{ri}^V = 5 \cdot K_{ri}^S \cdot (1 - \omega_V) \cdot \nu_V \quad (15)$$

– instantaneous rigidity of horizontal displacement

$$K_{ri}^H = 5 \cdot K_{ri}^S \cdot (1 - \omega_H) \cdot \nu_H \quad (16)$$

– instantaneous rigidity of rotation

$$K_{ri}^M = 4 \cdot r^2 \cdot K_{ri}^S \cdot (1 - \omega_M) \cdot \nu_M \quad (17)$$

The initial rigidity is a constant value

$$K_{ri}^S = \partial_{\delta_i} S_i(\delta_i = 0) \equiv a_s \cdot b_s \quad (18)$$

This rigidity is reduced by a function called the parameter of the degradation of rigidity. These parameters are computed as follows:

$$\omega_M \equiv e^{-b_s \frac{|M|}{M^III}} \quad (19)$$

$$\omega_V \equiv e^{-b_s \frac{|V|}{W^III}} \quad (20)$$

$$\omega_H \equiv e^{-b_s \frac{|H|}{W^III}} \quad (21)$$

5. REDUCTION COEFFICIENTS OF RIGIDITY

5.1. Definition

Since each parameter of rigidity degradation depends only on one component of force, we introduce reduction coefficients of rigidity.

$$\nu_V = \frac{\nu(V, 0, 0)}{\nu(V, H, M)} \quad (22)$$

$$\nu_H = \frac{u(0, H, 0)}{u(V, H, M)} \quad (23)$$

$$\nu_M = \frac{u(0, H, 0)}{u(V, H, M)} \quad (24)$$

where:

$\nu(V, 0, 0)$ – vertical displacement in the joint computed for exclusive action of the vertical force V ,

$\nu(V, H, M)$ – total vertical displacement computed for the mutual action of forces H , V and moment M ,

$u(0, H, 0)$ – horizontal displacement in the joint computed for the exclusive action of the horizontal force H ,

$u(V, H, M)$ – total horizontal displacement computed for the mutual action of the forces H , V and moment M ,

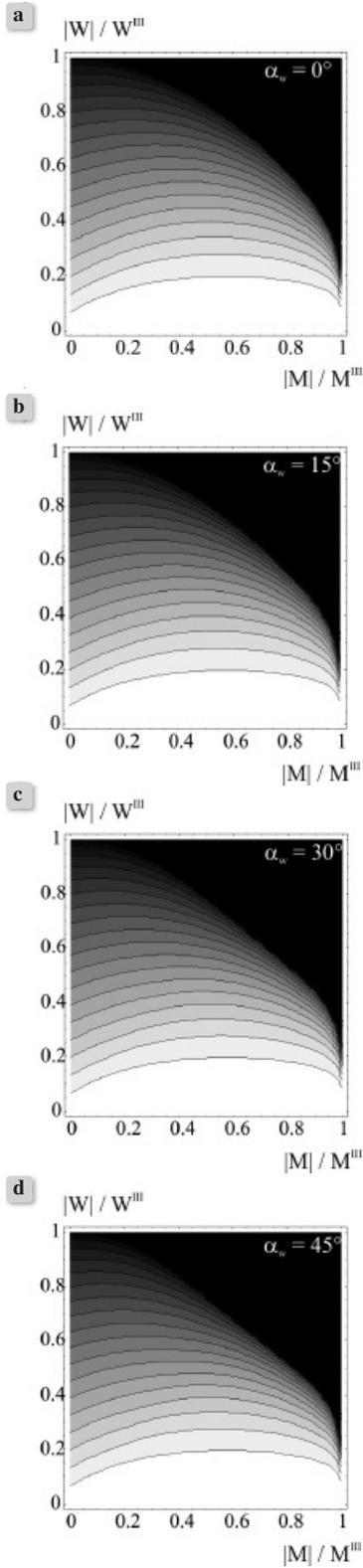


Figure 10. Contour maps of coefficients of reduction M for III case of the boundary state: when $\alpha_W = 0^\circ$, b) when $\alpha_W = 15^\circ$, c) when $\alpha_W = 30^\circ$, d) when $\alpha_W = 45^\circ$

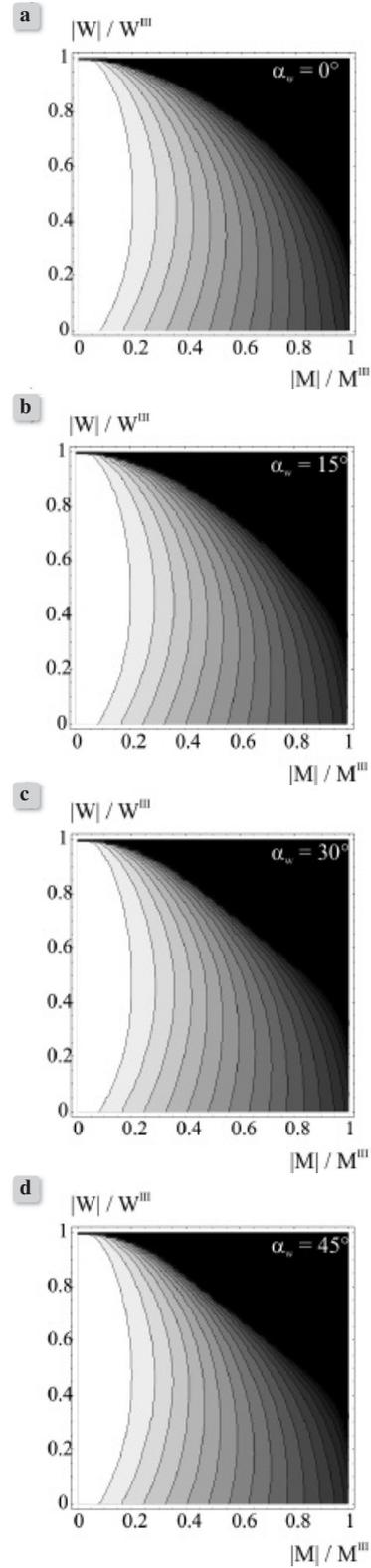


Figure 11. Contour maps of coefficients of reduction V for III case of the boundary state: when $\alpha_W = 0^\circ$, b) when $\alpha_W = 15^\circ$, c) when $\alpha_W = 30^\circ$, d) when $\alpha_W = 45^\circ$

$\phi(0,0,M)$ – angle of rotation in the joint computed for the exclusive action of the moment M ,

$\phi(V,H,M)$ – total angle of rotation computed for the mutual action of the forces H , V and moment M .

Reduction coefficients of rigidity have been computed for the considered 5-blind-bolt joint. Their diagrams are shown in Figs 10, 11 and 12. The diagrams in Figs 10 and 11 have been prepared for four different values of the angle α_W , see Fig. 1. They present the function of ν_M and ν_V , respectively. Figure 12 presents the function ν_H for one selected angle. The diagrams are plotted in the dimensionless domain.

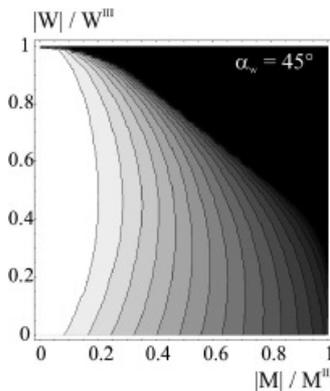


Figure 12. Contour maps of coefficients of reduction H for III case of the boundary state when $\alpha_W = 45^\circ$

5.2. Sensitivity analysis

Analyzing the diagram of the functions ν_M and ν_V (cf. Figs 10 and 12), we see that they are not very sensitive to variations of the angle α_W . Comparing Figs 11 and 12 we can state that the function ν_H is almost identical to ν_V .

5.3. Analysis of significance – and example

Analyzing these functions we see that the reduction of rigidities caused by the mutual influence of forces can be considerable. Let us consider a joint loaded with the shearing force (Eqn 13):

$$W = 0.4 \cdot W^{III} \equiv 0.4 \cdot 5 \cdot 58.58 \equiv 117.2 \text{ kN}$$

and the moment (Eqn 14):

$$M = 0.4 \cdot M^{III} \equiv 0.4 \cdot 4 \cdot 0.055 \cdot 58.58 \equiv 5.155 \text{ kN}$$

In the first case let us assume that the shearing force is sloped to wards the vertical axis at the angle

$\alpha_W = 45^\circ$ (see Figs 1 and 2b).

We can check, that the pair $W/W^{III} = 0.4$ and $M/M^{III} = 0.4$ lies within the area limited by the first limit curve (see Fig. 4d), so it is allowable from the engineering point of view.

We get the following forces in the blind bolts Fig. 1: $S_1 \equiv S_3 = 34.94 \text{ kN}$ (see Fig. 7d), $S_2 = 1.20 \text{ kN}$ (see Fig. 9d) and $S_4 = 42.34 \text{ kN} < S^I \equiv 48.6 \text{ kN}$ (see Eqn 10). It is worth noting that the bolt No. 2 is almost unloaded.

For the considered case we obtain the following values of the coefficients of reduction of rigidities: $\nu_M = 0.784$ (compare Fig. 10), $\nu_V \equiv \nu_H = 0.810$ (compare Figs 11 and 12).

In the second case we assume that the shearing acts along the vertical axis, $\alpha_W = 0^\circ$ (see Figs 1 and 2a).

We can check, that the pair $W/W^{III} = 0.4$ and $M/M^{III} = 0.4$ lies within the area limited by the first limit curve (see Fig 4a).

We get the following forces in the blind bolts (Fig. 1): $S_1 \equiv S_4 = 40.40 \text{ kN}$ (see Fig. 7a), $S_2 \equiv S_3 = 22.51 \text{ kN}$ (see Fig. 9a).

For the considered case we obtain the following values of the rigidities reduction coefficients: $\nu_M = 0.794$ (cf. Fig. 10a), $\nu_V = 0.820$ (cf. Fig. 11a). The coefficient ν_H is indeterminate when $\alpha_W = 0^\circ$, since the displacement $u \equiv 0$ (cf. Fig. 2a).

We can see that the reduction of rigidity is significant and therefore the mutual influence of forces should be taken into account.

6. CONCLUSIONS AND FINAL REMARKS

Experimentally verified nonlinear mathematical model presented in this paper can be used in the design and verification of multi-blind joints.

This verification can be supported by the limit curves, limit surfaces and diagrams presented in the paper. They can also be useful in the further design of experimental verification.

Provided analysis of rigidity shows that forces exert a mutual influence on instantaneous rigidities. The introduced and analyzed coefficients of rigidity reduction may be helpful.

Provided formulas for rigidity have already been applied in numerical analyze of 2D structures of thin-walled structures with flexible nodes, within the

Finite Element computer system [10]. Such an analysis requires an iterative approach, but has been successful. It has also been shown that the reductions of rigidities influence significantly displacements and internal forces.

Provided analysis can also be used in the further development of national and European design codes [11], [12].

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