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THE OVERVIEW OF FRACTURE MECHANICS MODELS FOR CONCRETE

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Abstract
Fracture mechanics of concrete is a complex matter still thoroughly researched from different angles. It is not an easy task to describe fracture process in concrete, as there are many factors affecting crack development and propagation. Practical applications of fracture mechanics could allow engineers to design concrete structures more effectively and safely. At the minimum, it could help estimate the "safe" period of time left before the unstable, dangerous crack propagation. This utilitarian goal was the reason for many researchers to invent numerous theoretical models in order to describe the crack occurrence better. However, dealing with various analytical problems was not a simple matter and thus existing models of fracture mechanics for concrete have different limitations. Over the years first fracture theories for concrete were reviewed repeatedly. All of these investigations lead to modifications of older models in order to overcome found drawbacks, which proved not to be an easy task. Recently, new approaches to fracture analyses seemed to produce promising results, like universal size effect law (USEL) or modified two parameter fracture model (MTPM) with alternative ways for evaluating fracture parameters. In the paper some of them will be discussed together with other fracture models, starting from some of the very first ones introduced for concrete, like fictitious crack model (FCM) and crack band model (CBM).

Keywords: **Fracture mechanics; Fracture of concrete; Fracture models; Cracking of concrete; Fracture mechanics of concrete.**

1. INTRODUCTION

Cracking of concrete is one of the most common flaws and problems in practical engineering. It can serve as an important signal that, for example, the construction is not working properly. It may also be a direct cause of structures' catastrophes even, if ignored.

It is not an easy task to describe fracture process in concrete, as there are many factors affecting crack development and propagation. In literature usually three main stages of the crack propagation are distinguished: crack initiation, stable crack development and unstable crack propagation. The third stage is especially problematic because it can cause a quick failure of a structural member. As a consequence, the need to research cracking processes in more depth has arisen in a form of a separate scientific field under the name of fracture mechanics. The start of its development is owed to A. A. Griffith [1]. In 1920 he formulated rules of an energetic model describing an element with an artificial flaw, simulating a naturally occurring phenomenon. The conclusion derived from this research was that a crack needs a certain amount of energy to lengthen, under a specific value of stress. Once the stress reaches a critical value, an unstable propagation occurs, followed by destruction of a structural member. His idea was a general one, using a mathematical model based on linear elasticity assumptions, and not focusing only on concrete. Even so the base of fracture mechanics was created. Next, after Griffith's theory, G.R. Irwin expanded fracture mechanics' research and introduced new parameters which are sometimes referred to as fracture criteria [2]. The most important one was a critical stress intensity factor K_{IC}, which was also the first concept of fracture toughness. Over fifty years after Griffith's first fracture theory a breakthrough for concrete structures was made when A. Hilleborg introduced his own model. It took into account properties of concrete as a material very different from steel, which was mainly researched material until then. After that many other theoretical models were proposed, the field of concrete fracture mechanics grew and new development is being conducted still, for example [3, 4]. Despite developing new solutions, some of the older models are still valid and applied in numerical and experimental research over recent years [5, 6, 7]. In this paper an analysis of main, selected models used to describe crack propagation in concrete will be carried out.

2. OVERVIEW OF CHOSEN THEORETICAL FRACTURE MECHANICS MODELS

2.1. Fictitious Crack Model (FCM) and crack band model (CBM)

In the following section two models will be discussed: the fictitious crack model and crack band model as they are mathematically and ideologically similar. Furthermore they struggle with coinciding problems. Differences between the models will be also pointed out.

The research of cracking begun with using knowledge derived from theory of linear elasticity. For concrete though it proved not to be the best approach, as it is not a perfectly brittle material. As a consequence, mathematical results showed stress singularity at the crack tip. That lead to the necessity of introducing modifications in a form of turning to nonlinear solutions. In case of fracture mechanics, the nonlinearity is related to fracture process zone (FPZ) which develops in front of a crack tip and replaces the erroneous idea of stress singularity. One of the first models for concrete was the fictitious crack model (FCM) introduced by Hilleborg et al. in 1976 [8]. It was based on the earlier work concerning metals which also fall under the category of materials behaving nonlinearly. The FPZ in the fictitious crack model was modeled in a linear form and the fracture process is characterized by a stress (in normal direction to the crack, σ) – displacement (δ) relationship in a softening range shown in the Fig. 1.

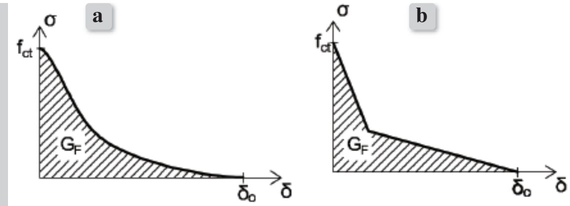

Figure 1.
Illustration of fracture parameters: a descending branch of stress-displacement curve (a) and a generalized bilinear softening function (b)

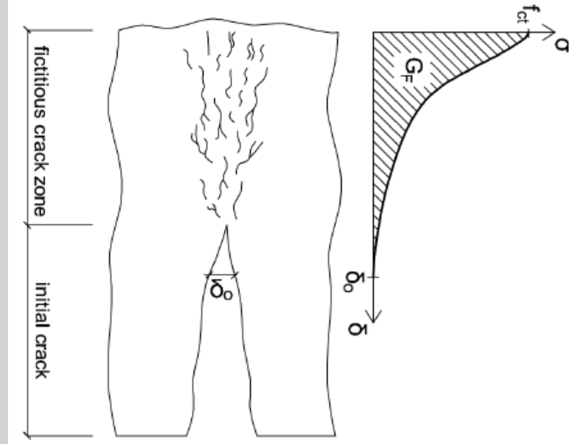

Figure 2.
The strain-softening phenomenon in the post-peak range and the fictitious crack model for concrete in tension [9]

That is the first of three main elements essential to the fictitious crack model (FCM). As the tensile stress decreases in the fracture process zone, the strain, such as the crack tip opening displacement, increases. This strain-softening occurrence for concrete in tension is presented in the Fig. 2 [9].

Two other elements that make up the core of FCM, labeled as the fracture mechanics parameters: the tensile strength limit f_{ct} and the fracture energy G_F, are also shown in Fig. 1. The concept of f_{ct} as one of the material properties is a well-known idea and its existence explains why there is no brittle fracture in concrete. On the other hand, G_F was a new value strictly connected to fracture mechanisms and it is described as the area under the curve shown in Fig. 1. The shape of the curve itself is also often called a fracture parameter and depends on the material. For example, for high strength concretes the initial part of the softening curve would be noticeably steeper compared to normal strength concretes. Fracture energy can be taken from Eq. 1 and it is understood as the area under the descending curve of σ-δ relationship.

$$G_F = \int_0^{\delta_o} \sigma(\delta) \mathrm{d}\delta \tag{1}$$

Another model that took into account the existence of FPZ was the crack band model (CBM) introduced by Bažant and Oh [10]. The main idea of CBM was for the deformations of the fracture process zone FPZ to be smeared over a band in front of the main crack's tip. That produced an additional material parameter – a fixed value of crack band width (w_c). It seemed as a more realistic approach from the fictitious crack model, seeing as in concrete there usually was a lot of microcracks distributed along some part

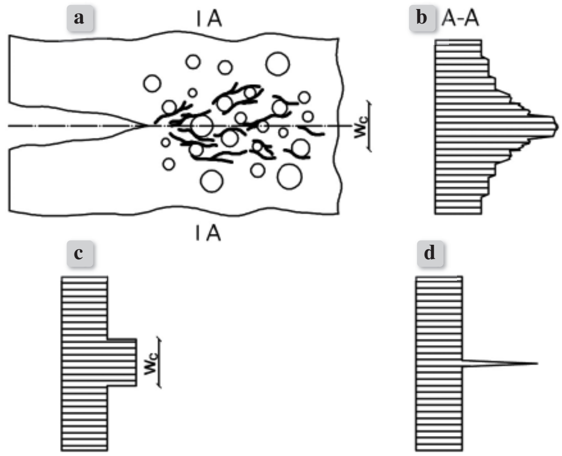

Figure 3.
The visualization of the crack morphology (a), the model of strain distribution in the fracture process zone and its vicinity (b), the crack band model (CBM) concept with the crack band width wc (c) and the fictitious crack model (FCM) concept for comparison (d)

of the material in front of the crack tip that later coalesced into bigger cracks. Mathematically though the fictitious crack model was a special case of a crack band model, if an assumption was made that w_c was infinitely small ($w_c \rightarrow 0$). It was also proven that these two models were basically equivalent in terms of the results they gave.

Even if the fracture process zone (FPZ) and its significant size in concrete was taken into consideration in the models, there were still a few points at issue. The first one would be the model of the crack itself in the fictitious crack model (FCM). There is a possibility of a singular crack existing and propagating but the linear shape of it is quite a significant simplification. The more natural path of propagation of such a defect is a torturous one. The cause of that is the heterogeneity of concrete. Because the aggregate and the cement matrix have different strength properties, the matrix being the weaker one, it's only natural that the crack will develop avoiding the stronger aggregate particles. However, there are exceptions to this rule when considering high-strength and ultra high-performance concretes in which the matrix's strength tends to be higher in comparison with the aggregate's one. In such cases there is a possibility of a more straight crack propagation, cutting also through the grains. The matter of the fictitious crack model being an one-axial is also an issue to point out. In reality most common constructions work in a three-axial stress state. On the other hand, it is possible to take into account the three-dimensional stresses in terms of fracture, for example if the linear crack model is not used [11]. Also the crack analysis becomes more complicated in reinforced concrete, where the presence of steel can cause cracks to appear parallely. In such case, the fictitious crack model (FCM) does not present realistic results.

Looking at the crack band model, in which theoretically the crack form is shaped more realistically in a smeared manner (Fig. 3), the question about w_c appears.

The width of a crack band, also known as a FPZ width, should be fixed for numerical analysis' purposes but various research results could not seem to agree on the definitive value of this parameter. The most popular thesis stated that the width of FPZ was dependent on the maximum aggregate's size (D_{max}). This dependence was researched over the years [12-14] and varies for different reasons, for example the type of concrete or the type of a tested member, it's shape and size. There are also some findings that did not confirm w_c – D_{max} dependency at all [15]. Another concept to consider would be the stability of crack band during the crack propagation. The general idea in crack band model is that of a fixed value but if it is really not dynamic at all during the progressing of fracture and the course of loading. One thing which is quite certain though is that crack band's width has a considerable impact on numerical results. The problem is discussed wider for instance in [16]. One undeniable conclusion is that the crack band width is a relevant parameter of the crack band model (CBM) used commonly in numerical simulations and as such it requires a finite and consistent rules of applicability.

The final issue with discussed models is the size effect which could be observed for the nominal stress at maximum load. Fracture energy G_F was proven to depend on the geometry of tested concrete members, in terms of both size and shape, as well as the aggregate size. Recent research shows that there are other factors impacting the fracture energy [17], such as: water-cement ratio or type of aggregate.

That fact effectively restrained the possibilities of these first crack models for concrete to be utilitarian in their design applications. The need for appropriate size effect law to be used, in order to avoid this problem and extrapolate the value of fracture energy for infinite size was a continuous struggle and an element of research, e.g. [18, 19]. This issue will be discussed in more depth in paragraph 3 of the paper.

2.2. Two Parameter Fracture Model of Jenq and Shah (TPFM)

One of the first special fracture models for concrete with no direct usage of stress – displacement curvature was the two-parameter fracture model (TPFM) proposed by Jenq and Shah in 1985 [20]. Instead, linear elastic fracture mechanics' (LEFM) elements were modified in order to take into consideration slow crack growth which is a nonlinear phenomenon in concrete. Fracture parameters were introduced as key elements to this model: the critical stress intensity factor KIC and the critical value of the crack tip opening displacement $CTOD_c$. Both of them required to be measured experimentally in Mode I conditions, in a three – point bending test of a notched concrete member.

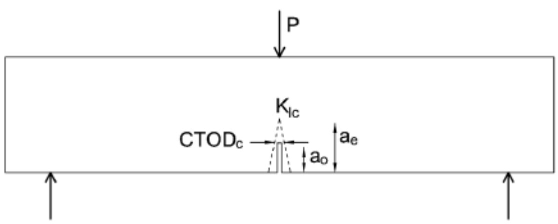

Figure 4.
A notched beam in a three-point bending test with parameters of Jenq – Shah model

However, an issue appears to emerge in relations to these fracture parameters such as the dependency on composition of concrete mix. It has been proven that water-cement ratio has an impact on fracture parameters [21, 22]. Fly ash and slag were also researched in relation to that matter [23].

Other values crucial to the two-parameter fracture model TPFM, connected to aforementioned main two parameters, need to be considered as well (Fig. 4): the effective crack length a_e and the length of an existing, artificial notch a_0. At the tip of the effective crack, corresponding to the maximum load, K_{IC} should be measured. The width of the notch introduced for the experimental purposes gives the value of $CTOD_c$. However, the calculations for all those parameters are based on the LEFM equations and were proven to be geometry – dependent. Sundara Raya Iyengar et al. [24], based on an earlier attempts of other researchers, proposed an alternative, graphical method of determining critical stress intensity factor, as well as the critical value of the crack tip opening displacement. Graphical solutions that could produce fracture mechanics' parameters in a simpler way would be a meaningful aid. On the other hand, it should be remembered they are also derived from LEFM equations which in case of concrete can lead to various mathematical issues. Finally, Jenq – Shah model was based on mode I only, leaving room and necessity for further analysis of mode II, mode III and mixed modes, which would presumably lead to an improvement of understanding fracture in concrete structure on a more realistic and practical level.

The experimental method of measuring itself is burdened with some difficulties, such as possibility of inaccurate calculating of K_{IC} and $CTOD_c$ because of crack propagation after the peak load. This topic was raised for example by D. C. Jansen et al. [25]. Therefore the authors proposed a different, modified testing procedure for the two-parameter fracture model – the focal point method. It was divided into two sub-methods: the focal point method I and II. Both of them were graphic – based, similarly to the solution proposed in [24]. Focal Point methods serve as a simulation of the closed – loop testing in a way for determining unloading compliances, which are later used in a standard two-parameter fracture model (TPFM) to calculate K_{IC} and $CTOD_c$. In method I the idea is to find a centroid of a triangle constructed from three lines going through points of three, initial peak loads. The centroid then acts as a foundation for determining unloading compliance C_{uc}. In the second method two lines from the first loading and first unloading are required and their intersection point provides also a solution for defining C_{uc}. These methods seem to give more realistic values of two basic fracture parameters but still only for testing under mode I conditions.

2.3. Modified Two Parameter Fracture Model (MTPM)

With regard to the crack model itself there were attempts made to take into consideration the tortuous path that the crack may follow. In that case the reason generally may be twofold. The kinked cracking may occur as a result of a difference in strength of cement matrix and aggregate or as a consequence of mixed mode loading. A. Carpinteri et al. [26] carried out a series of test focusing on the second aspect concerning the deflection of a crack from its supposed, linear path (Fig. 5). The modified two parameter model's core consists of mathematical changes in formulae in a form of introducing a deflection angle θ, for computing the value of fracture parameters.

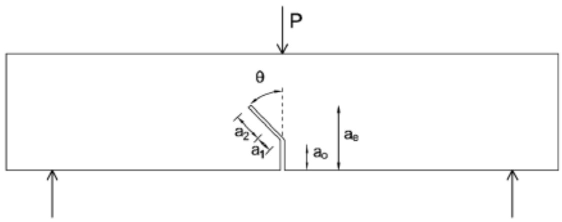

Figure 5.
Theoretical basics of modified two parameter fracture model: the deflection (kinking) angle θ, effective critical crack length a_e, a_1 and a_2 together form the kinked crack branch [27]

The theoretical background for the tests was implementation of additional virtual forces perpendicular to the notch. These forces along with the basic vertical, loading force in a three-point bending together created conditions of mix mode loading. For obtaining the critical stress intensity factor $K_{(I+II)c}$ the effective crack length is necessary [27]:

$$a_e = a_o + a_1 + a_2 \tag{2}$$

Where: a_o – initial crack length, $a_1 = 0.3a_o$ and a_2 can be calculated through an iterative procedure. In case the a_2 proves to be negative, the effective crack length consists only of a_o and a_1.

Although the starting point of described modified two parameter fracture model's (MTPM) analysis involved bovine cortical bones, quite recent research adapted and implemented it into fracture of concrete. Some similarities may be observed with regard to the general quasi – brittle nature of both materials but the structure of bone tissue with many elements of various shape and mechanical characteristics is fairly different from concrete structure, which consists of only two basic parts – the matrix and the aggregate. However, the aim to take account of the crack behavior under mixed mode loading would be a valuable input in concrete fracture mechanics. The investigation in [27] shows promising results based on fiber-reinforced concrete members.

2.4. Double – K Fracture Model (DKFM)

Double-K model was based on a criterion proposed by Xu and Reinhardt [28]. As the name suggests, it consists of two separate stress intensity factors. The idea behind this combined form of a stress intensity factor, which in itself was known and considered before [2, 29], was to propose a tool for estimating crack propagation in a structure in a twofold way. The first one represents the initial cracking toughness K_{Ic}^{ini} and the second one – unstable fracture toughness K_{Ic}^{un}. The initial stress intensity factor is especially significant, as describing the phase of initial crack propagation was not attempted before. Other models already mentioned are related to the last phase only – unstable cracking. For three-point bending tests the values of K_{Ic}^{ini} and K_{Ic}^{un} can be determined from the following formula based on the test data [28]:

$$K_{Ic} = 1{,}5\frac{PS}{BD^2}\sqrt{a}F(\alpha) \tag{3}$$

where: P – the applied load, S – length of the beam, B – thickness of the beam, D – depth of the beam, a – the crack length, F(α) – function dependent on the a/D proportion.

Depending on which values of P and a are put into the above formula, the K_{Ic}^{ini} and K_{Ic}^{un} can be obtained.

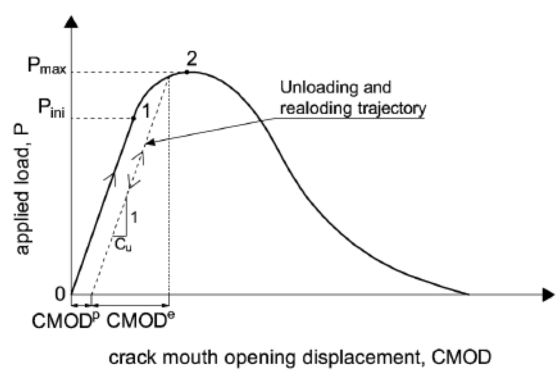

Figure 6.
An example of a P – CMOD curve for the purposes of calculating the double – K fracture parameters, where: C_u – the unloading compliance, P_{ini} – initial cracking load, $CMOD^p$ – plastic crack mouth opening, $CMOD^e$ – elastic crack mouth opening. From point 0 to 1 the fracture process can be characterized as elastic and there is no further propagation of the initial crack, at point 2 maximum load P_{max} is achieved

For evaluating initial stress cracking toughness, the value of initial cracking load P_{ini} is needed. One of the possible ways of determining P_{ini} is through the use of P-CMOD curve (Fig. 6) but it is not an easy task [30]. Hence the analytical way of evaluating initial stress intensity factor was proposed:

$$K_{Ic}^{ini} = K_{Ic}^{un} - K_{Ic}^{c} \tag{4}$$

The unstable fracture toughness K_{Ic}^{un} can be simply obtained from Eq. 3. K_{Ic}^{c} is the stress intensity factor due to cohesive force and can be found through the use of the distribution function of the cohesive stress along the fictitious crack zone at the peak load (Fig. 7). It is important to note that the shape of cohesive stress distribution is assumed to be linear between the initial notch tip to the tip of effective crack extension.

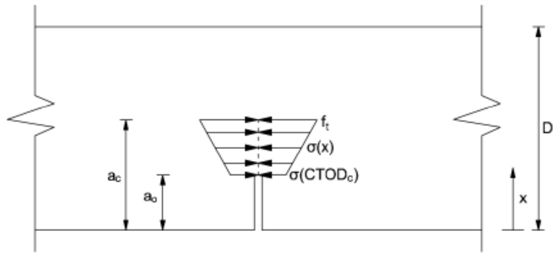

Figure 7.
The cohesive stress distribution σ(x) in the fictitious crack zone at the peak load; a_o – initial crack length, a_c – critical crack length, f_t – tensile strength of concrete

However, a problem can occur with the analytical solution, as it is a simplified method of evaluating fracture toughness and the possibility of discrepancy between mathematically obtained results and practical use exists. Next to the mentioned two types of fracture toughness gives another, subsidiary fracture parameter – critical crack tip opening displacement $CTOD_c$ [30], as in the two parameter model of Jenq and Shah. Unlike the K_{Ic}^{ini} and K_{Ic}^{un} parameters, experimental research showed the size-dependence of $CTOD_c$.

In order to investigate the evaluating procedure of double – K parameters in wider spectrum, besides three-point bending tests, Xu and Reinhardt carried out a series of experiments on compact tension specimens, as well as wedge splitting specimens [31]. The results revealed that for large CT members K_{Ic}^{ini} and K_{Ic}^{un} show no dependency on size. These conclusions are logically somewhat similar to the idea of "extrapolation to infinity" about which Bažant et al. talks in his research of size effect. Further discussion about this topic will be carried out in the next part of the paper. On the other hand, for the small WS specimens K_{Ic}^{ini} and K_{Ic}^{un} were found to increase in value as the size of the tested members grew.

Wang et al. [32] tested specimens of different compressive strengths for initial and unstable cracking toughness with consistent results. They proposed different, size-independent empirical formulas from Xu and Reinhardt, for obtaining K_{Ic}^{ini} and K_{Ic}^{un}:

$$K_{Ic}^{ini} = 0{,}108(f_c)^{0{,}497} \tag{5}$$

$$K_{Ic}^{un} = 0{,}447(f_c)^{0{,}341} \tag{6}$$

in which f_c is the cube compressive strength of concrete.

Further research is still ongoing and throughout the recent years was added to the investigation of double-K fracture parameters [33–36].

3. SIZE EFFECT IN FRACTURE OF CONCRETE

Size effect is a problem associated with many theories and models of fracture mechanics (Fig. 8) and at the same time it presents an obstacle in universal and practical use of fracture parameters. Because of the importance of this challenge, this part of the paper will be dedicated to it as a separate paragraph.

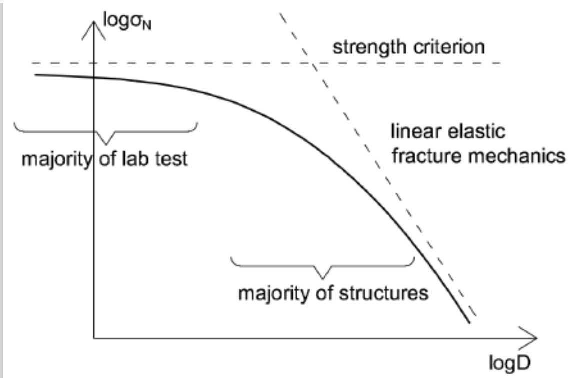

Figure 8.
Size effects for geometrically similar structures on a bi-logarithmic scale with strength criterion and LEFM asymptotes (dashed lines). Solid arched line shows the transitional behavior of concrete between different types of size effects [37]

This problem was observed very early and the idea of finding a golden mean surfaced a few years after introducing first fracture models for concrete. Many laboratory tests and analytical investigations were carried out [38–40]. A lot of research was carried out by Bažant [41–44]. He analyzed size-effect occurring in RILEM recommendations and observed that the fracture energy obtained through these recommendations was strongly dependant on size and also on the notch length of the members [41]. Experimental studies have been carried out in order to find size-independent G_f by other scientists and compare its values obtained through various methods [45]. Similar studies from various angles are continuously carried out.

Bažant based his first investigations on the size effect law which was proposed for nominal stress. The first evaluation of the nominal strength, however, was also size-dependant:

$$\sigma_N = c_N \cdot \frac{P_{max}}{b \cdot d} \tag{7}$$

where: c_N – after Bažant, a general coefficient for convenience [41], P_{max} – ultimate load, b – thickness of the specimen, d – characteristic dimension of the specimen.

Another form of formula used for fracture of concrete, devised also by Bažant [42], for nominal stress, was as shown below:

$$\sigma_N = \frac{B \cdot f_t}{\sqrt{1 + \frac{d}{\lambda_o d_a}}} \tag{8}$$

or alternatively with the use of fracture characteristics:

$$\sigma_N = c_N \cdot \sqrt{\frac{E' G_f}{g'(\alpha_o) c_f + g(\alpha_o) d}} \tag{9}$$

where: f_t – tensile strength, B and λ_o – empirical constants, d_a – the maximum aggregate size, d – characteristic dimension of the specimen, E' – Young's modulus for plane stress or plane strain, G_f – fracture energy, $g(\alpha_o)$ – dimensionless energy release function, $\alpha_o = a_o/d$, a_o – initial notch depth, c_f – the fracture zone length at maximum load.

In literature the λ_o is also replaced by do and named as the transitional structure size [43] or the brittleness number β [41]. Other researchers also made attempts to calculate and assign brittleness numbers to investigated materials, e.g. Carpinteri [44] or Gustafsson and Hilleborg [46], in order to predict possible fracture behaviour. The empirical constant could be characterized as another fracture parameter and in [44] the role of it is compared to the character of Reynolds number. In [46] the measure of brittleness took a form of a characteristic length l_{ch} which is proportional to the fracture energy.

One of the main theory principles dealing with size dependency is the existence of two types of size effects for fracture occurring in quasi-brittle materials such as concrete: type I (deterministic) and type II (energetic). The formula mentioned above (Eq. 8) is connected with type II size effect or in general it is called the size effect law of Bažant. This occurs when the specimens are geometrically similar, with a considerably large depth of the notch in comparison to the characteristic dimension D. Type I size effect formula is as follows [43]:

$$\sigma_N = f_r^{\infty} \cdot [1 + \frac{r \cdot d_b}{d + l_p}]^{\frac{1}{r}} \tag{10}$$

Where: f_r – nominal strength extrapolated to infinity for very large specimens, d_b – boundary layer length, r – empirical constant, l_p – characteristic length of the material.

This type of size effect is caused by the randomness of material strength and it originated from Weibull's theory [47]. To simplify briefly, type I size effect occurs when the crack length is equal to 0 and type II size effect is observed for deep notches.

However, there are cases where the fracture behaviour in a concrete member falls neither strictly under type I, nor type II size effect and instead it is a transitional variation of the two. That was a direct cause of introducing a combined law under the name of universal size effect law (USEL). In [43] Hoover et al. raise an additional point of statistical element and thus the USEL has two forms. Other research on a general size effect law was carried out also [48, 49]. A significant analysis was performed by Hoover and Bažant [50], in which the authors gathered enough experimental material to properly verify the applicability of USEL. These comprehensive tests lead to defining an empirical parameter which describes size effect law in terms of transition between type I and II. The question of geometry independence of this parameter remains still unanswered and needs to be verified in further tests.

Other researchers also focused on the problem of size effect in fracture of concrete over the course of many years. Carpinteri et al. considered the fractal nature of cracking in concrete and its input to the size effect [51, 52]. It resulted in introducing the multifractal scaling law (MFSL) which analytically is given by the following equation:

$$\sigma_N = (A + \frac{B}{d})^{\frac{1}{2}} \tag{11}$$

where: σ_N – the nominal tensile strength, A and B – the constants related to the physical dimensions of the square of a stress and the square of a stress-intensity factor respectively, both to be determined from a non-linear least-squares numerical algorithm, d – characteristic structural size.

The idea behind this approach is to consider renormalized fracture parameters (such as the tensile strength and the fracture energy) given by the analysis of the observation scales. At the normally used, macroscopic scale, the fracture energy is dissipated over the nominal area. In the fractal sense, the scale would be progressively enlarged, leading to the decrease of the impact that microcracks have on the mechanical behaviour of concrete. At the same time the fracture energy becomes a renormalized, scale-invariant fracture parameter as a part of a more orderly multifractal scaling concept, in a form of a linear relationship (Fig. 9a). Analogical reasoning can be applied to the tensile stress (Fig. 9b).

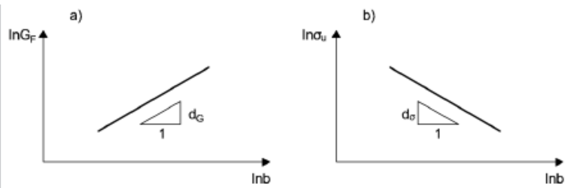

Figure 9.
The effects of multiscaling fractal law on a bi-logarithmic scale: a) for the fracture energy, b) for the tensile stress; b – the characteristic reference size, d_G – the fractional increment of the energy dissipation, d_σ – the fractional decrement caused by the analyzed disorder

Another approach to the size effect was taken by Duan and Hu in a form of the boundary effect concept [53]. The idea was to associate the size-dependant fracture energy and strength with the member boundary – crack tip relation (Fig. 10).

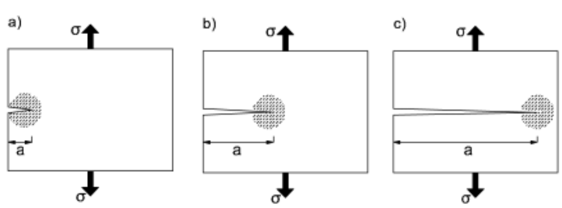

Figure 10.
The member boundary – crack tip relation in the boundary effect concept: a) the case of a "short crack", when the crack's tip is close to the edge, in other words front boundary, b) the crack tip is remote from both front and back boundaries and the linear elasticity's rules applies, c) the case of a "short ligament" when the crack tip is close to the back boundary. The hatched area corresponds to the fracture process zone

In short, the localization of the fracture process zone will directly affect the fracture parameters. If the crack length is short (a→0), the maximum stress will reach the tensile strength of concrete as the crack itself will remain shielded before specimen failure. On the other hand, if the crack length is significant and the fracture process zone is localized in the middle of a specimen and far away from the boundaries layers, the strength criterion will follow the rules of linear elastic fracture mechanics LEFM.

Further investigations lead to the development of an asymptotic boundary effect model [54] which produced the asymptotic solution as follows:

$$\sigma_N = \frac{f_t}{\sqrt{1+\frac{a}{a_\infty}}} \tag{12}$$

where: f_t – the tensile stress, a – the crack length, a_∞ – the reference crack length depending on the stress intensity factor, at the intersection of two asymptotic limits.

However, the equation mentioned above is applicable only for the large specimens so the LEFM conditions are possible.

4. CONCLUSIONS

The broad scale analysis of the professional literature presented in the paper has proven that fracture mechanics of concrete is worth dealing with. Each of the discussed models has its limitations. The two-parameter fracture model and size-effect model produce very similar results but at the same time there are other parameters which differ from one fracture model to another. For example, the fracture energy G_f – Hilleborg's model and the size-effect model produce disparate values of G_f. Additionally, other several unknowns and points are still being discussed by scientists.

When describing fracture processes in concrete, a great deal depends on concrete strength, type of aggregate, concrete age or geometry of a structural member. Proven size-dependency of many fracture parameters derived from the discussed models in the paper is one of the main problems concerning the theory of fracture mechanics, as well as the key to find universal laws for evaluating fracture parameters for all types of structural members. Even in case of the size effects models there are still questions about the range of their applicability, especially in terms of non-linear conditions. Initial notches which are artificially introduced into the researched member in three-point bending tests, used by all the models, are not always a good imitation of the actual cracks that could exist internally in concrete structures. Majority of theoretical range in all the discussed models depends on the linear elastic fracture mechanics, whereas the lack of linear behavior of fracture in concrete is very apparent. Another point is the utilization of beams in the three-point bending tests mainly in the models. That kind of laboratory environment will not always depict realistically the actual concrete behaviour in terms of cracking. It could be worthwhile to conduct more research on cylinders, which are the type of sample that can be extracted from the actual existing concrete structures.

In spite of a variety of tests which are still being conducted in the field of fracture mechanics, it can be concluded that both fracture parameters of concrete as well as fracture models need further investigations. In particular the influence of the depth of the notch and the maximum aggregate size on fracture parameters are worth to deal with.

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