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HYPERBOLIC PARABOLOID (HP) PANTOGRAPHIC STRUCTURE WITH LINER SCISSORS

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Abstract
A pantograph is a foldable structure which consists of scissor link units. A unit consists of two bars elements which are capable of rotating about their intermediate pivot node. The pantographic structures are generally utilized in flat (like roof), cylindrical (like barrel), and spherical (like dome) deployable structure and they are not used in anticlastic structure like hyperbolic parabolic (HP) structure. The HP surface may form when a convex parabolic goes on the length of concave ones with the same curvature. On the other hand, the hyperbolic surface can be constructed using two families of mutually skew lines in which the lines in each family are parallel to a common plane, but not to each other. In this paper, the creation of HP surface with pantographic structure is presented. The creation of a HP pantographic structure is demonstrated with the use of three methods including: a) two border scissors; b) four border scissors; c) All-scissor HP Pantographic Structures. Finally, the proposed methods have been compared.

Streszczenie
Pantograf jest konstrukcją składaną, która składa się z połączonych ze sobą ramion. Jednostka składa się z dwóch ramion, które mogą się obracać wokół swojego pośredniego węzła obrotowego. Struktury pantograficzne są na ogół stosowane w takich konstrukcjach struktur płaszczyznowych (np. dach), struktur cylindrycznych (np. zbiornik walcowy) i sferycznych (np. kopuła), w których możliwe jest przemieszczanie elementów struktury. Struktury pantograficzne nie są stosowane w konstrukcjach ukształtowanych z wykorzystaniem paraboloidy hiperbolicznej, których przemieszczenie elementów struktury wydaje się być niemożliwe. Hiperboloida paraboliczna może być ukształtowana poprzez przesunięcie paraboli po krzywej kierującej w postaci paraboli. Powierzchnia hiperboloidy parabolicznej może być również skonstruowana poprzez złożenie dwóch rodzin prostych skośnych równoległych do płaszczyzn kierujących tych rodzin. W artykule przedstawiono tworzenie powierzchni HP ze strukturą pantograficzną. Tworzenie struktury pantograficznej HP przedstawiono trzema metodami, takimi jak: a) metodą pary ramion skrajnych; b) metodą czterech ramion skrajnych; c) metodą wieloramiennej struktury pantograficznej. Na zakończenie porównano zaproponowane metody.

Keywords: **Deployable Structure; Hyperbolic Paraboloid (HP) Surface; Liner Scissors; Pantographic; Scissor-like element (SLE).**

1. INTRODUCTION

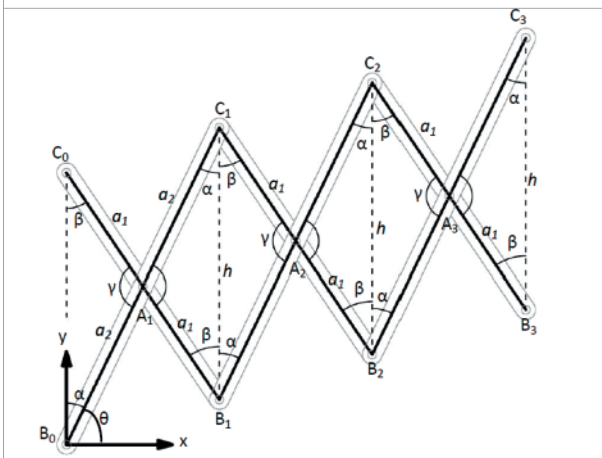
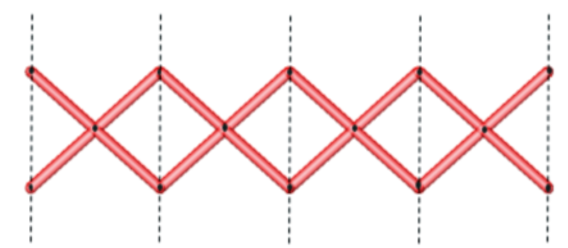
Throughout history, people have tried to construct flexible buildings that are adaptive to varying conditions and requirements. The main goal of transformable architecture is designing adaptive spaces and building facades through mechanical structure application which is the main component to such buildings [1]. Deployable structures are capable of transformation in an independent way. The most common transformations include transforming from a closed and contracted form to an expanded and unfolded form [2]. In categorization of transformable structures, pantographic structures belong to the category of moving structures in which the structural system can change its shape of geometry through a certain mechanism without any elastic transformation in the members [3].

In the early 1960s, Spanish architect, Emilio Pérez Piñero employed a scissor mechanism in which every member had three pivotal joints with one joint on each end of the scissor and another one at the center. Once the two ends of the scissor mechanism convert, the centers of pivots diverge and the structure fully unfolds in a flat pattern [4]. Felix Escrig and John Valcarcel modified Piñero's work. They developed new planes and space grids in order to achieve various geometries. In particular, they focused on achieving various geometries using identical members and different connective elements [5]. Escrig, Sanchez and Varcarcel examined two-way spherical pantographic structures by dividing the sphere's surface. This two-way grid requires members such as intersecting bars or cables for the steadiness of the structures while it is deployed; otherwise, lack of triangular pattern causes results in unsteadiness [6]. The possibility and conditions of the relationship between the span and member dimensions were first described by Escrig and Valcarcel. However, their most important work is rigid plate roofing element [7]. In addition to constructing several models, Escrig has designed a retractable roof with pantographic mechanism for a swimming pool in Seville. The design included two rhombic grid structures with spherical curve [8]. Deployable pantographic columns are linear structures that are made of translational and circular units that are investigated by Raskin. He focused on the behavior of pantographic structures as the mechanism for expansion process and stabilized expanded forms by adding boundary conditions ([9], [10]). Under the direction of Sergio Pellegrino, a research group called Deployable Structures Laboratory at Cambridge University was formed in 1990 for conducting studies on deployable structures. One of the designs of this group was a deployable pantographic ring structure used as a mast for space antennas. This structure included translational moduli in the inner and outer rings that are connected by module through circular geometry [11].

Chuck Hoberman is an influential designer focusing on the mechanism of pantographs. He tried to develop some early geometrical forms by using pantographic mechanism applications. Developing horn, icosahedral, spherical, and geodesic-dome forms are examples of those designed using pantographic mechanism [12]. However, Hoberman's most important accomplishment in transformable pantographic mechanism design is the simple angulated element. This element includes two identical, angulated members joined together by a complex connection and it is the basis for the new generation of transformable structures [13]. Using these angulated members, Hoberman created Iris deployable dome and Hoberman Sphere [12]. Sergio Pellegrino and Zhang Yu of Deployable Structures Lab (DS), Cambridge University created Multi angulated, pantographic structures based on Hoberman's work. A full set of these structures is capable of inward radial deploying and it can produce any plan forms [14]. Matthias Rippmann and Werner Sobek's research is a study by the Institute for Lightweight Structures and Conceptual Design (ILEK) at the University of Stuttgart. This study is based on the new scissor-like element (SLE). This element has multiple joints that allow the member to be connected at different points. By changing the location of the points, different shapes can be produced [15].

One of the problems in scissor-like structures is weakness in covering various forms. Forms that have been possible so far using these structures include flat structures such as flat ceilings, structures with one curve like arch and cylindrical structures and structures with two same-orientation curves such as spherical structures. The main feature of hyperbolic parabolid (saddle) structures is that it is double curved in two different orientations. Due to the visual and to some extent performance complexity of the form, it is impossible to use pantographic structures for these types of forms and to deploy them. The purpose of the present study is making hyperbolic paraboloid deployable forms using pantographic structures and to make it feasible to apply these structures in different forms.

Table 1.
Categorization of Translational Pantographic Structures

Members with Different length	Members with Fixed length
Two members have different length Joint is at the middle point Joint pivots are parallel to each other	All members have equal length (2a) Joint is at the middle point Joint pivots are parallel to each other
	
[16]	[18]

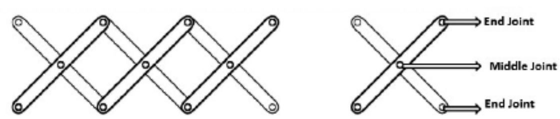


Figure 1.
Scissor-like element and structure (Author)

2. PANTOGRAPHIC STRUCTURES

A pantographic structure consists of a number of scissor-like elements (Fig. 1) and each scissor-like element is made of two rigid members (e.g. bars). These elements are connected by a joint on a point on their span. The joints allow members to rotate about the main axis and limit rotation and movement in other axes. Scissor-like elements are connected by end joints and allow the system's movement in the desired orientation and power transfer. In addition, with the movement of scissors alongside each other and changes in their shape and geometry, the whole system is deployed or folded.

Geometry of pantographic structures basically depends on the geometry of scissor elements. The whole system changed by changing the length of each member of the bars, though the changes were not equal. Also, the changes occurred as a result of changing in the direction of the connection pivot on scissor-like elements or as a result of changing in the position of elements in the space [16]. In order to understand scissor-like structures, simple formal conditions in relation to member span should first be understood. In terms of scissor-like elements, pantographic structures are divided into three categories of translational, curved and angulated pantographic structures [17].

Translational pantographic structures can be constructed by a simple straight-line translation and without any angular changes in the whole system. The main principle in this type of pantographic structure is that all lines that connect joints should be parallel to each other. Two translational structures are formed with respect to member span and placement of middle joints. The categorizations of these types of structures are presented in Table 1.

Curved pantographic structures are created by changing the placement of the middle joint from the central point to the side point. These types of structures may have two different forms. This type of system can be deployed either as a circle (with one center) or free curve and presented in Table 2.

Unlike translational and curved pantographic structures, members in angulated pantographic structures are not straight-lined and they have an angle like β at the joint position. Scissor structures with angulated members can have two different forms. This type of system can either have one angle or deploy with multiple angles. These angulated elements are used for deployable structures that are in form of a compact configuration and are capable of folding and unfolding outwardly and these conditions are not possible

Table 2.
Categorization of Curved Pantographic Structures

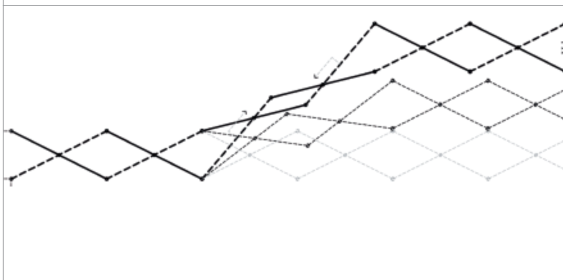
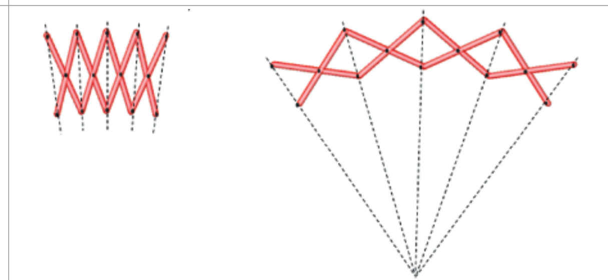
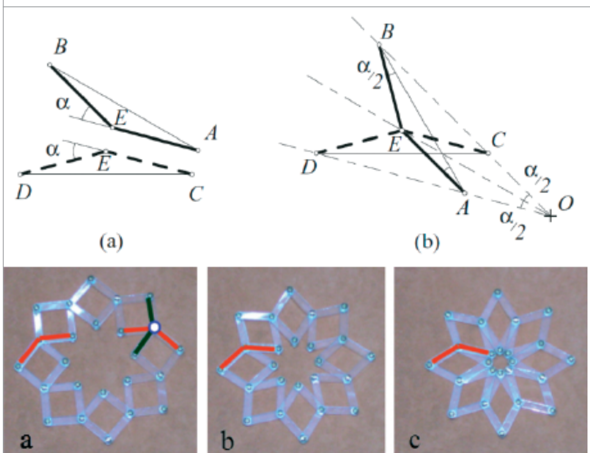
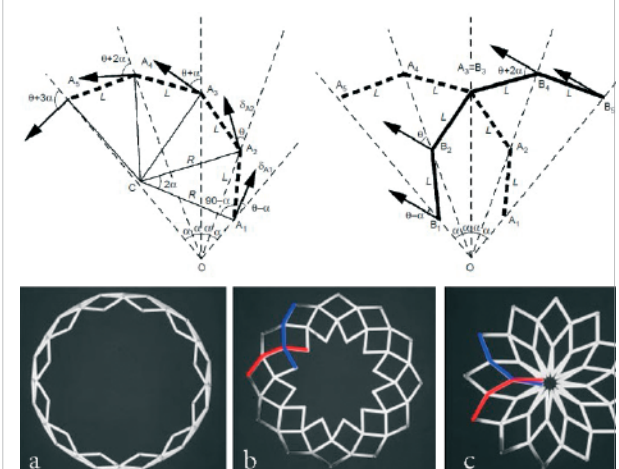
Members With Free Geometry	Members With Circular Geometry
Unique size of each scissor-like modulus All pivots don't joint at one point	Like part of an arch that contracted and deployed All pivots joint at one point
	
[19]	[17]

Table 3.
Categorization of Angulated Pantographic Structures

Angulated	Multi-Angulated
Two rigid half-bras with equal length and α angle The angle α has remained permanent during the deployment process	More angles compared to angulated elements Includes multiple half-bars connected with identical angles equal to α
	
[20]	[21]

for pantographic structures with circular geometry due to their linear deployment. See Table 3.

3. GEOMETRY OF HYPERBOLIC PARABOLOID (HP) SURFACES

The generation of surfaces can be conducted by geometrical or non-geometrical techniques. Geometrical surfaces can be divided into surfaces including revolution, translational surfaces, ruled surfaces and free-form surfaces. Revolution surfaces are generated by rotatation of a curve around an axis. The surface created via the revolution method is a synclastic surface (Fig. 2-a). Translational surfaces are formed by sliding a plane curve (generator) along with another plane curve (directrix). In this process, the orientation of the sliding curve remains constant. Contingent on the curvatures of the generator and the directrix, surfaces such as synclastic and anticlastic can be created (Fig. 2-b). Ruled surfaces are created by sliding the two ends of a straight line on their own curve, while remaining parallel to a prescribed direction or plane. Ruled surfaces are formed via straight lines only; a ruled surface is anticlastic (Fig. 2-c) [22]. Hyperbolic parabolid surfaces can be generated out

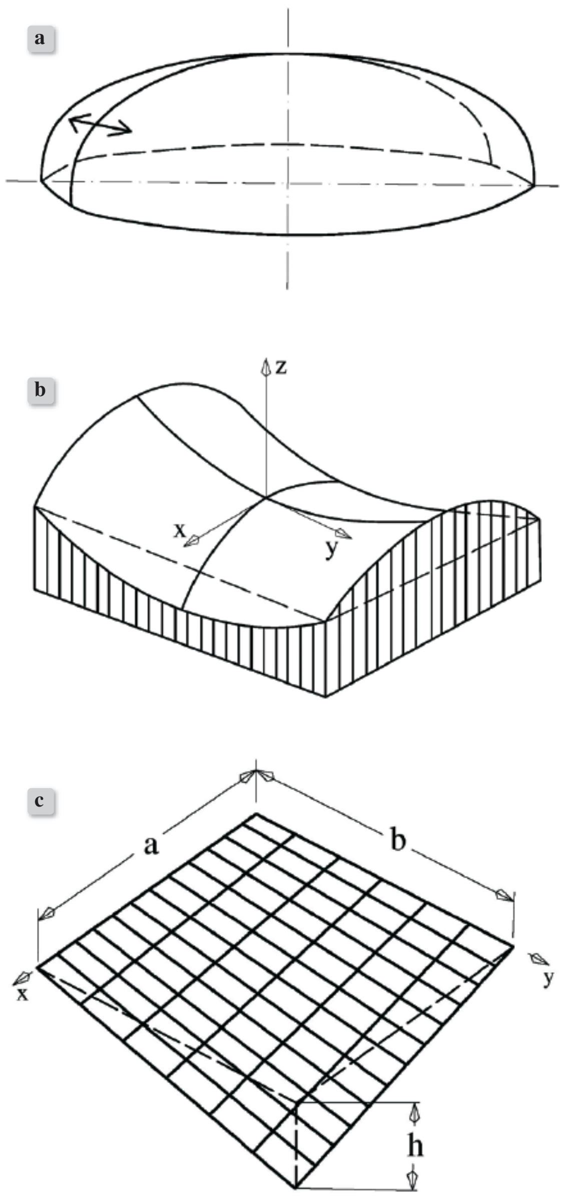


Figure 2.
Geometrical Surface Generation, a – Surfaces of Revolution, b – Translational Surface, c – Ruled Surface [23]

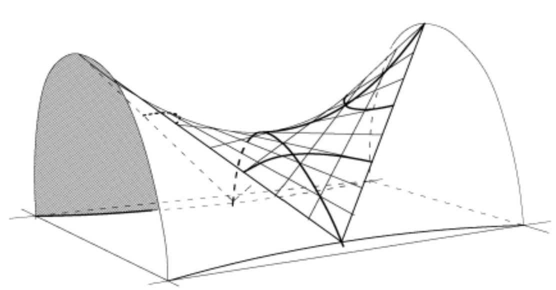


Figure 3.
Surface of HP [23]

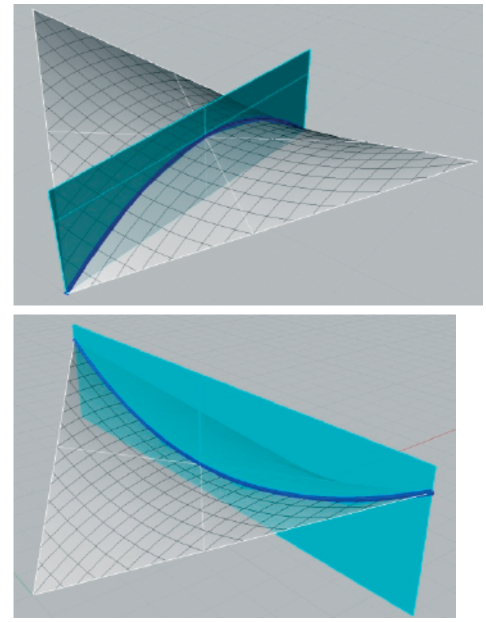


Figure 4.
Main Curvature Lines in HP Surfaces (Author)

of translational and ruled surfaces.

HP surface is obtained by transferring a parabola with downward orientation to another parabola with upward orientation [24]. This surface is similar to a saddle. Its horizontal planes have two branches separate from a hyperbolic curve, whereas its main vertical planes are paraboloid (Fig. 3).

HP surfaces are curved downward in lateral direction and curved upward in longitudinal direction [22]. If these surfaces are intersected with a perpendicular, rotating-plane, not only does its curvature change in value but also in orientation (Fig. 4). If the perpendicular intersecting plane rotates about its own axis, form curvature gradually changes from positive to negative values, then values return to positive [25]. Therefore, curvature should be zero at both directions. HP surfaces do not have similar orientations in all intersections. Usually, they have zero curvature at two certain directions along which there are straight lines on its surface [26]. These surfaces are not deployable [22]. Main curvature lines in HP surfaces are illustrated in Fig. 4.

Surfaces that provide zero curvature at more than two orientations on one point are easily constructed. HP surfaces have two series of straight lines at their surface. Surfaces that are created when the two ends of a segment slide on two separate curves are called ruled surfaces [24]. When the two curves are segments standing diagonally, the ruled surface creates a HP (Fig. 5).

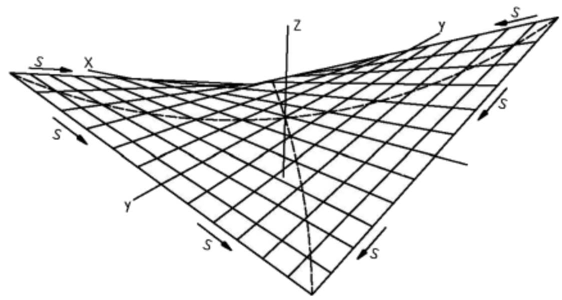


Figure 5.
Ruled HP Form [27]

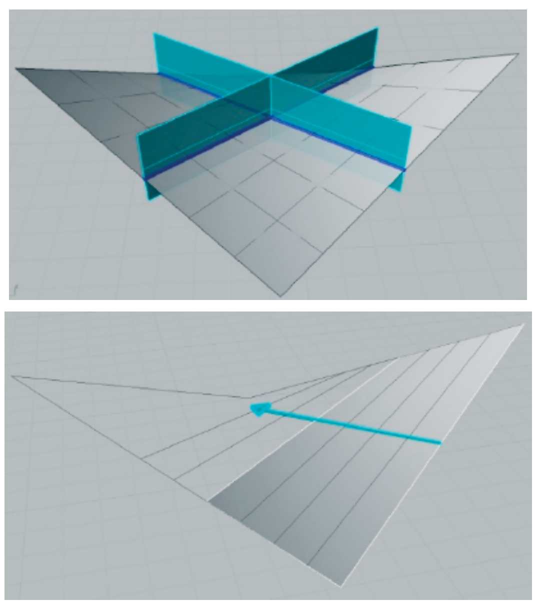


Figure 6.
Straight-line Generators of HP Surfaces (Author)

Like all ruled surfaces, the curvature of HP surface reaches zero from two direction. For hyperbolic paraboloid surface, however, this direction has a single orientation for all points. This means that all perpendicular intersections parallel to these two directions are straight lines that are called generative lines [25]. Moreover, HP surfaces have two series of straight lines that are on their surface [23]. Therefore, this surface may be obtained as a result of the movement of a line along two diagonal lines in space. In this paper, this method has been used to create HP forms (Fig. 6).

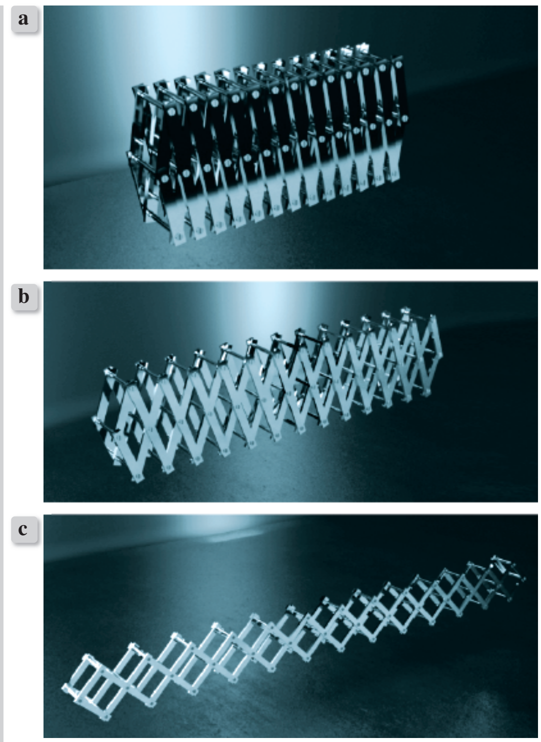


Figure 7.
Details of Linear Border Scissors Employed
a) Contracted State
b) Half-deployed State
c) Deployed State; (Author)

4. HYPERBOLIC PARABOLOID PANTOGRAPHIC STRUCTURE WITH LINER SCISSORS

In this section, using computer modelling and laboratory models, it is attempted to investigate generation of HP pantographic structures by relying on three methods of HP pantographic structures with two border scissors, HP pantographic structures with four border scissors, All-scissor HP Pantographic Structures. Furthermore, a table presenting a comparison between the aforementioned methods is provided at the end of this section. In the present paper, Rhinoceros software has been used for the simulations of all modelling procedures.

4.1. HP Pantographic Structures with Two Border Scissors

First, by relying on two linear border pantographic structures, a HP form is contracted and deployed. HP surfaces can be generated by two different methods. In the first method, using two curves with different curvatures and with the sliding and movement of one

Figure 8.
Deployment and Contraction Stages of the Computer Modelling of two Border Scissors with Cable Grid Covering
a) Contracted State b) Half-deployed State c) Deployed State; (Author)

Figure 9.
Deployment and Contraction Stages of the Computer Modelling of Two Border Scissors with Membrane Covering
a) Contracted State b) Half-deployed State c) Deployed State; (Author)

Figure 10.
Deployment and Contraction Stages of the Laboratory Model of two Border Scissors with Cable Grid Covering
a) Contracted State b) Half-deployed State c) Deployed State; (Author)

curve along the other one a HP surface is created. In the second method, it is possible to create the mentioned form by sliding a carve (generator) along with other carves (directrix). In this paper, the second method is applied for creating HP forms. According to the definition of pantographic structures, deployable HP forms are created using two translational pantographic structures with fixed (or variable), inversely-oriented span and are used as the path and a fixed element (which can be cables and hard bars or coverings such as membranes) that move in line with the two scissors' nodes. These two pantographic structures are identical where opposite nodes are connected with some elements or cables or any kind of covering. With the movement of the scissors along the movement path, the structure is deployed and the desired form is created. When contracting, by moving along the same line and in inverse path, scissors cause the form to be folded. In the contracted state, cable grids are folded as a series of straight lines in the corners of the structure. An alternative is to use membrane covering in which case the surface is fully covered and when contracting, the membrane covering is folded at the top of the surface and in a linear form. In Fig. 7 the details of translational scissors with identical border spans and the border used, in Fig. 8 deployment and folding stages of the computer model with cable grid covering, in Fig. 9 deployment and folding stages of the computer model with membrane covering, and in Fig. 10 deployment and folding stages of the laboratory model with cable grid covering are shown. Dimension of the model was

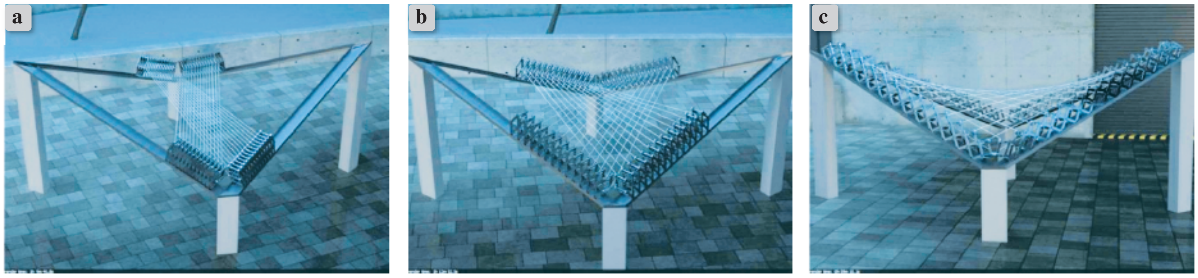


Figure 11.
Deployment and Folding Stages of the Computer Model of Border Four-Scissor with Cable Grid Covering
a) Contracted State b) Half-deployed State c) Deployed State; (Author)

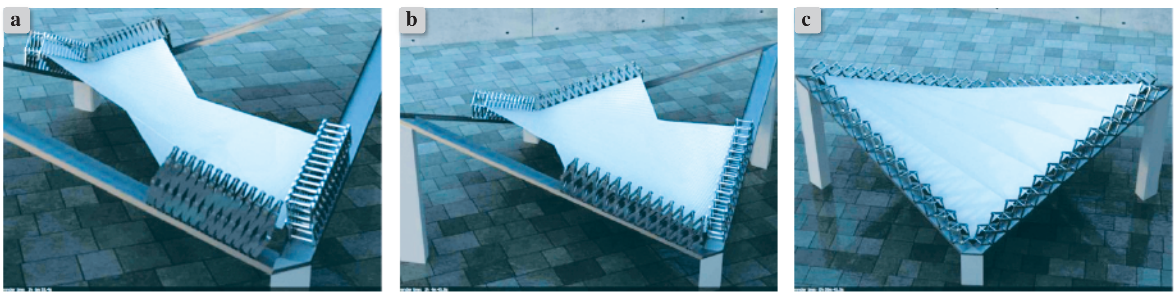


Figure 12.
Deployment and Contraction Stages of the Computer Model of a Border Four-Scissor with Membrane Covering
a) Contracted State b) Half-deployed State c) Deployed State; (Author)

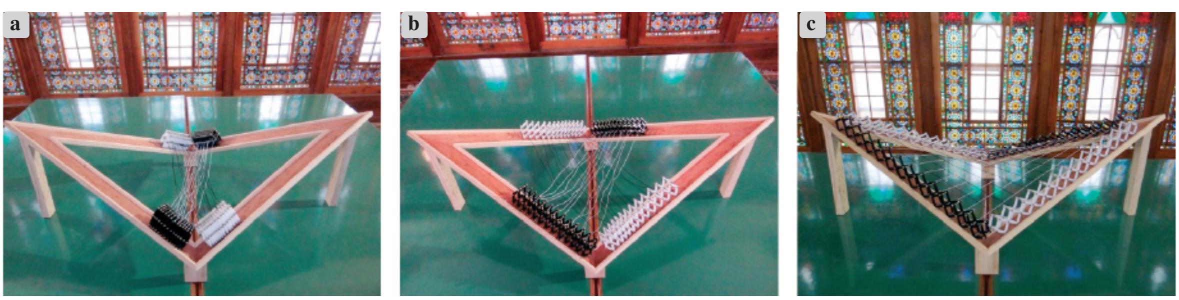


Figure 13.
Deployment and Contraction Stages of the Laboratory Model of a Border Four-Scissor with Cable Grid Covering
a) Contracted State b) Half-deployed State c) Deployed State; (Author)

70×70 and height of its legs were 5 and 30 cm. Number of the employed modules was 13. The materials employed for making parts of the scissor were Plexiglas glass, which were cut using CNC machine. Size of each part of modules in the scissors was 7×7 cm.

4.2. HP Pantographic Structures with Four Border Scissors

HP forms are ruled forms i.e. such forms are generated by straight lines. This structure is the middle stage between two-scissor and all-scissor structures. Placement of scissors includes two translational scissors with identical (variable) spans which constitutes the path as in the border two-scissor structures, as well as two other translational scissors with identical (or variable) spans that connect the beginning and end of the mentioned scissors. In addition, a cable grid or a membrane is reciprocally connected to the mentioned scissors' nodes. The contraction position of the structure in this system is the bottom corner of the complex and at two points. The system starts to deploy once the scissors move. The movement of the four is in such a way that it begins at bottom points and continues as far as the scissors are fully deployed and when they reach the top point of the form. Finally, when the system is fully deployed, scissor

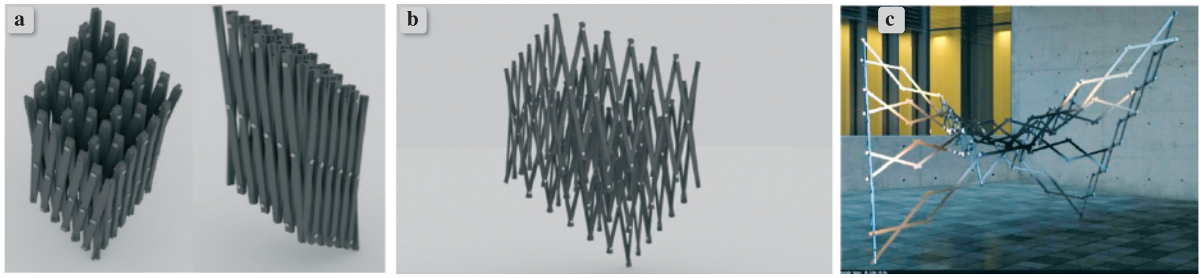


Figure 14.
Deployment and Contraction Stages of the Computer Model of the All-scissor Structure
a) Contracted State b) Half-deployed State c) Deployed State; (Author)

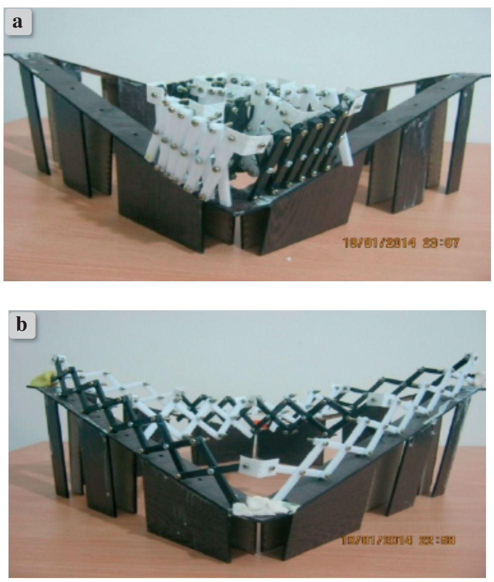


Figure 15.
Deployment and Contraction Stages of the Laboratory Mode of All-scissor form
a) Contracted State b) Deployed State; (Author)

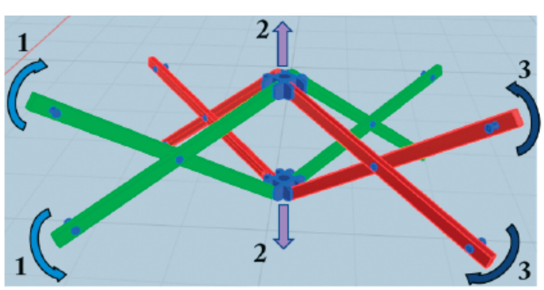


Figure 16.
Details of Deployment and Contraction Stages of the Computer Model of the All-scissor Structure (Author)

ends meet and form is complete. The onset of system's contraction is in inverse direction of the deployment path and while diverging, the scissors move to the corners at the top and the whole complex returns to the two end points and the system is folded. Deployment and folding stages of the computer model with cable and membrane covering and of laboratory model are shown in Fig. 11 to Fig. 13, respectively. Dimension of the model was 70×70 and heights of its legs were 5 and 30 cm. Number of the employed modules was 13. The materials employed for making parts of the scissor were Plexiglas, which were cut using CNC machine. Size of each part of modules in the scissors was 7×7 cm.

4.3. All-scissor HP Pantographic Structures

According to the definitions of HP formations, we may create a HP form by sliding the two ends of a straight line on their own curve, while remaining parallel to a prescribed direction or plane. In all-scissor HP pantographic structures all members are scissors such that both the path line and the generative line of a translational pantographic structure have different length. Thus, a two-way grid of translational pantographic structures with different length is created that allows deployment and contraction of the HP form at all points. The number of scissors varies depending on number of moduli and other requirements and there will be no other limitations regarding the deployment and contraction of HP forms. Considering that the system is a grid, it has different deployment points and it can be deployed or contracted at all connective points. At the connection points of scissors' grid, the rotation of one scissor translates into the rotation of the scissor perpendicular to it and these rotations cause the scissors to move. With the moving of perpendicular scissors on each other, the entire grid moves in both directions that are connected and the whole complex is contracted or deployed depending on the type of rotation (Fig. 16). Fig. 14 presents deployment and folding stages of the computer model and Fig. 15 shows the deployment and folding stages of the laboratory model.

Table 4.
Comparative Table for HP Pantographic Structures (Author)

Characteristic	Type of Structure		
	HP Pantographic Structures with two Border Scissors	HP Pantographic Structures with four Border Scissors	All-Scissor HP Pantographic Structures
Location of Contraction	At the End of Either Generative Axes	At the Lowest Point of the Form, Middle of the Structure	At the Connection Point of Scissor
Type of Covering	Membrane	Membrane	Membrane
Type of Covering span	Small	Average	Large
Mechanism Complexity	Low	Mild	High
Performance Type	Simple	Mild	Complex
Minimum Number of Operators	Two	Four	Two
Transportation	Easy	Mild	Difficult
Damage-Resistance	Low	Average	High
Number of Members	Low	Average	High
Covering Control	Low	Average	High

Dimension of the model was 30×30 and heights of its legs were 2.5 and 15 cm. Number of the employed modules was 6. The materials employed for making parts of the scissors were Plexiglas, which were cut using CNC machine. Size of each part of modules in the SLE was 7×7 cm.

Table 4 compares characteristics of two-scissor, four-scissor and all-scissor pantographic structures.

5. CONCLUSION

In this paper, the procedure for creating HP pantographic structures is shown in three different methods. In the first method, two border scissors are used for creating a structure which has a simple mechanism and is easy to control. In the second method, four ruled border scissors are used with each scissor having members of equal length as in the translational two-scissor method. The salient feature of this system is deployment at accessible points. In the last method, the entire main lines of the form are of translational pantographic system with members of different length which allow for the structure to deploy and fold at any connection. The main characteristic of all three methods is using translational pantographic structures that have linear movement. These linear structures have been used for deploying and folding HP forms that have two different curvatures at two different directions. Creation of different forms and diverse coverage in scissor-like elements has always been one of the most serious challenges for users of these structures. Future studies can be carried out with the aim of scissor-like structures with free forms and different methods of coverage for such structures.

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