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PARAMETRIC SHAPING OF CONSISTENT ARCHITECTURAL FORMS FOR BUILDINGS ROOFED WITH CORRUGATED SHELL SHEETING

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Abstract
The paper presents a method for a parametric shaping of innovative building free forms covered with corrugated steel shell roofs and flat glass elevation walls. Each shell roof is made of plane steel sheets folded in one direction and transformed elastically into spatial shape. The obtained shapes can be really diversified sectors of various ruled surfaces. Therefore there is a great need for creating such a method that leads to a fine integration of the shapes of a free form shell roof and plane oblique elevation walls for each building free form. Thus, it was elaborated the innovative parametric method, whose algorithm allows for seeking attractive and consistent architectural free forms by means of computer programs. A relatively small set of initial relevant parameters enables the designer intuitively to affect the attractiveness of the designed free forms. Some special attention is paid to selected fine proportions of these geometrical parameters leading to the really attractive architectural free forms.

Streszczenie
Artykuł prezentuje metodę parametrycznego kształtowania innowacyjnych swobodnych form budynków przekrytych fałdowymi powłokowymi dachami stalowymi oraz płasko-ściennymi elewacyjnymi ścianami szklanymi. Każdy dach powłokowy wykonany jest z płaskich stalowych arkuszy fałdowanych jedno-kierunkowo oraz przekształcanych sprężyście do postaci przestrzennych. Metoda prowadzi do zadowalającej integracji kształtów dachu powłokowego i płaskich, pochylonych ścian elewacyjnych swobodnej formy budynku. Algorytm metody pozwala w prosty sposób poszukiwać atrakcyjne, swobodne formy architektoniczne także przy pomocy programów komputerowych Stosunkowo mały zbiór odpowiednio dobranych parametrów początkowych pozwala projektantowi w sposób intuicyjny wpływać na atrakcyjność projektowanych form swobodnych. Specjalną uwagę zwrócono na wybrane proporcje pomiędzy tymi parametrami geometrycznymi prowadzące do zbudowania szczególnie atrakcyjnych form swobodnych.

Keywords: **Free forms; Building shell forms; Corrugated roofs; Parametric shaping; Consistent architectural forms; Integrated transformed shapes; Folded steel sheets.**

1. INTRODUCTION

A certain specific type of innovative building free forms is considered. A relatively great freedom in geometrical shaping these forms results from two important assumptions. Firstly, their roofs are composed of nominally plane standardized folded steel sheets transformed elastically into shell shapes [1]. Secondly, their elevations can be inclined to the vertical at almost any required angle [2, 3].

The folded sheets are connected to each other with their longitudinal edges into a strip called the corrugated sheeting, which after spreading over roof directrices adapts its shape in a certain range to the form and mutual position of the directrices [1], Fig. 1, 2.

The directrices support the sheeting transversally to the shell fold directions.

Figure 1.
The experimental shell sheeting supported by curved directrices [1]

Figure 2.
The shell sheeting supported by straight and curved directrices by Adam Reichhart [4]

Because a freedom of transverse width increments of each shell fold in a sheeting is ensured during its shape transformation, the longitudinal axis of each shell fold is identical with its twist axis [2]. It is called the neutral axis of the shell fold, and stays straight during and after the shape transformation. Therefore such shell sheetings can be modelled with ruled surfaces [5, 6, 7], Fig. 3.

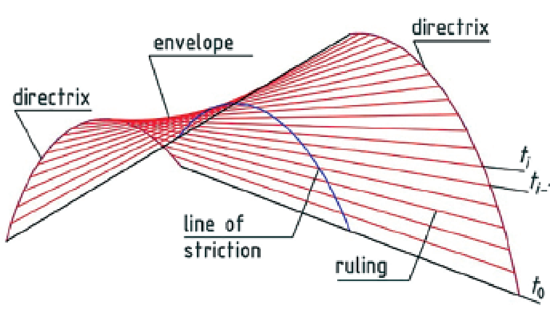

Figure 3.
Properties of ruled surface central sectors modelling a corrugated shell sheetings [8]

It is possible to achieve a large diversity of such shell roof free forms depending on diversification of various shapes and mutual locations of the roof directrices [9]. However, important restrictions in shaping the roof free forms result from diversified stiffnesses of their shell folds such as longitudinal, transverse flexural, torsional and shear ones [2, 10]. As a result of the above restrictions, the relevant shells can be built [2, 11, 12]. Sectors of ruled surfaces are used as the models for the shells because the neutral axes of the subsequent shell folds are skew straight lines [1, 13].

The freedom of shaping plane elevation walls results from the possibility of diversification of their inclination to the vertical. However, an important restriction relies on imposing a condition that the roof directrices have to be contained in the elevation wall planes [3, 9], Fig. 4.

Figure 4.
The free form roof by Adam Reichhart

2. STATE OF THE ART

Shell roofs made up of the flat sheets folded in one direction and transformed into spatial shapes are investigated by Gergely, Banavalkar and Parker in [14]. All roof shells considered there are of parabolic hyperboloid shapes, so they are called hypars. Gioncu and Petcu also have analysed such hyperbolic paraboloid shapes in more complex configurations in [15]. Bryan and Davis describe such shells in a more complex way in [16]. The shells are shallow, so their structures are often used.

Adam Reichhart has elaborated simple algorithms for a method of geometrical and strength shaping shell roof free forms made up of freely deformed single-corrugated steel sheeting [4], Figs. 2, 4. He can be regarded as the precursor of shaping such corrugated free form roofs. However, its algorithms are very reduced. Firstly, the shell folds are modelled with very simple central sectors of right ruled paraboloids. Secondly, he has only built models for free twisted folds. He has used the above models for shaping folds undergoing big bending and bending-twist shape deformations. So his analyses have very reduced character. This is way, the number of variables describing shapes of shell folds he has exploited is insufficient. Thirdly, he has not imposed any condition that the contraction of each shell fold has to pass through a half on the fold's length, which is necessary to obtain the really free deformed shell fold. Fourthly, his strength calculations are linear as for small deformations and strains, while the deformations here are intentionally big and, in addition, the folds are of open thin-walled profiles.

Jacek Abramczyk has elaborated more accurate innovative method of geometrical shaping such corrugated transformed shells [8, 16]. It is based on diversified stiffnesses of the folds affecting orthotropic properties of the considered shell sheetings [1, 12]. He has also extended the method to shaping free forms of entire buildings [2, 3] and their structures [2, 9].

The modified method exploits the so-called control compositions Γ, that are tetrahedrons of a specific type, Figs. 5, 6.

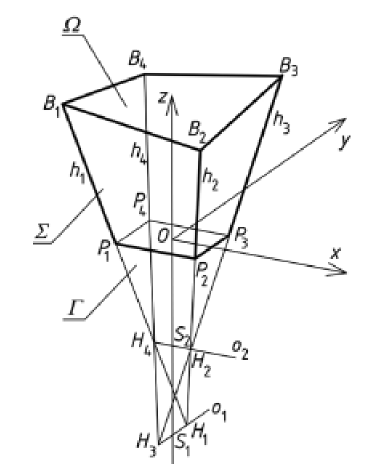

Figure 5.
A model Σ for a building free form with straight roof directrices B_1B_2 and B_3B_4 [2]

Four planes of each control composition Γ contain four tetragons $<P_1P_2B_1B_2>$, $<P_2P_3B_2B_3>$, $<P_3P_4B_3B_4>$ and $<P_1P_4B_1B_4>$ modelling plane elevation walls of a building. The sector Ω of a warped surface limited by the closed spatial quadrangle $B = <B_1B_2B_3B_4>$ is a model of a shell roof of the building. Thus, a sum of the above four elevation tetragons and a sector Ω is a free form Σ modelling this building. So, Ω belongs to Σ and is contained in the inside of Γ.

Each control composition Γ can be created by means of the following initial data, Fig. 5:

a) six variables describing the location of four vertices H_i (for $i = 1$ to 4) in the local co-ordinate system $[x, y, z]$ and four straight lines h_i containing four side edges P_iB_i of Σ, where two of these variables describe the distance and inclination angle between two skew straight lines o_1 and o_2 as well as four variables prescribe the location of each vertex H_i on o_1 or o_2 towards S_1 or S_2; if we assume, for a moment, that a new position of the above co-ordinate system origin is at S_2, $x = o_2$ and y $\parallel o_1$ as well as o_1 and o_2 are perpendicular to each other, then the above vertices have the following co-ordinates: $H_2(x_{H2}, 0, 0)$, $H_4(x_{H4}, 0, 0)$, $H_1(0, y_{H1}, z_{H1})$ and $H_3(0, y_{H3}, z_{H3} = z_{H1})$, so there are 5 independent variables; however, when o_1 and o_2 are not perpendicular to each other, then we have the 6th independent variable;
b) three variables describing the locations of three points $P_i \in h_i$ defining a plane base of the free form Σ;
c) four variables prescribing the locations of the vertices B_i belonging to h_i and the border line B of Ω.

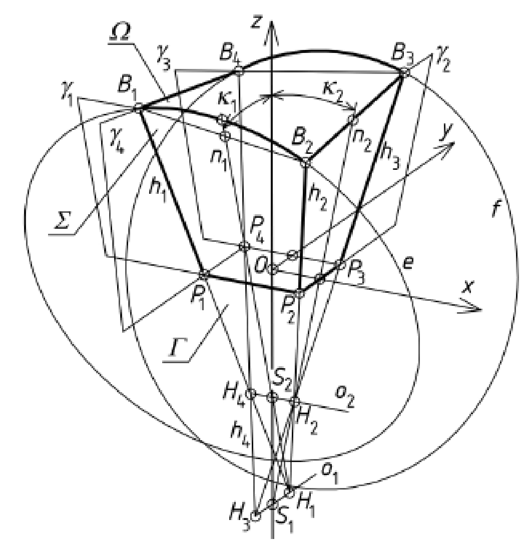

Figure 6.
A model Σ for a building free form with curved roof directrices *e* and *f* [6]

To summarize, each pair of the vertices H_i determines one of the side edges h_i, and two pairs of proper vertices H_i determine two axes $o_1(H_1, H_3)$ and $o_2(H_2, H_4)$ of Γ.

However, other types of sets of the variables defining the control compositions can be used [5, 10]. Three basic ways of creating the control compositions Γ and free forms Σ are proposed in the author's monograph [2].

The author of the present paper has also proposed a new innovative method of shaping free forms of building structures that is based either on the reference surfaces [7, 8] or demands connecting many individual free forms with their elevation walls [3]. In these cases, many individual smooth roof shells close to each other have to be located regularly in three-dimensional space to form one ribbed or discontinuous roof shell structure [17, 18], Fig. 7.

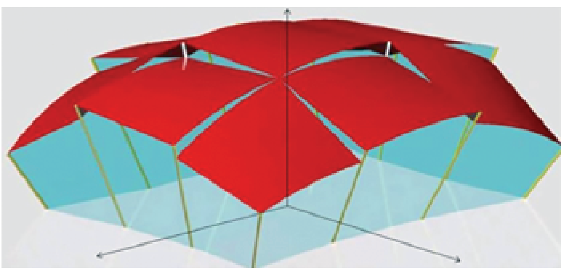

Figure 7.
Discontinuous free form shell structure [17]

An initial action enabling to create consistent architectural free forms is proposed in the precursory papers [18, 19, 20] by Aleksandra Prokopska and Jacek Abramczyk, Fig. 8. In the above works, there are presented various configurations of complete and complex free forms covered with corrugated steel roof shell sheeting and plane glass elevation walls.

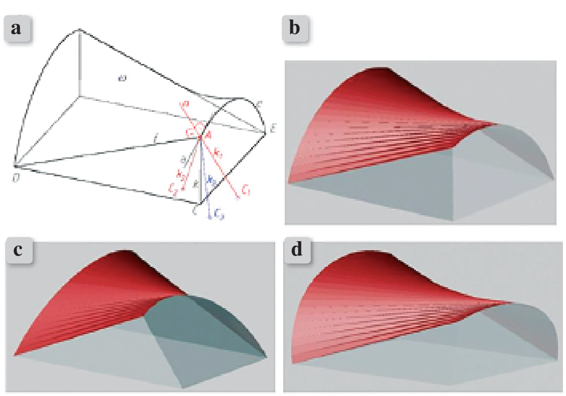

Figure 8.
On shaping integrated and consistent architectural free forms [18]

3. AIMS

The main aim is to present utilization of a method of parametric shaping of the complete innovative building free forms in terms of searching for attractive and consistent architectural free forms. Therefore a relatively small set of parameters and especially their chosen proportions enabling intuitively to affect the attractiveness of the required architectural free forms of entire buildings are analysed. The important feature of the method is that it allows to obtain a fine integration of shapes of a shell roof and plane oblique elevation walls.

The parametric shaping of the attractive free forms is presented in a few examples. Some special attention is paid to such selected values of these parameters that lead to really attractive innovative architectural forms. In the author's opinion, even some ranges of these values proposed in the present work may allow for achieving a great number of attractive free forms characterized by fine proportions of their edges and areas.

On the basis of the parametric description of the free forms used by the method, simple prototype computer programs were written in AutoLISP – the language of programming AutoCAD to seek for optimal proportions leading to diversified and really innovative architectural forms.

4. CONCEPT OF THE METHOD

Division of the method's algorithm into three main stages seems to be obvious. At the first stage, a reference polygon Δ is constructed with the help of four z-axis-symmetric auxiliary rectangles Δ_1, Δ_2, Δ_{1m} and Δ_{2m} characterized by a very specific mutual position on a common plane, Fig. 9. The reference polygon defines transverse dimensions of the roof and elevations of a determined free form. Here, the polygon Δ is defined by a set of four parameters $a = |C_1C_2|$, $c = |C_5C_6|$, $d = |C_4C_5|$ and $h = |C_1C_6|$ expressing the lengths of edges of the above two rectangles. In the examples below the parameter $b = |C_2C_3| = h - d$ is used to define important proportions considered in the further part of the paper.

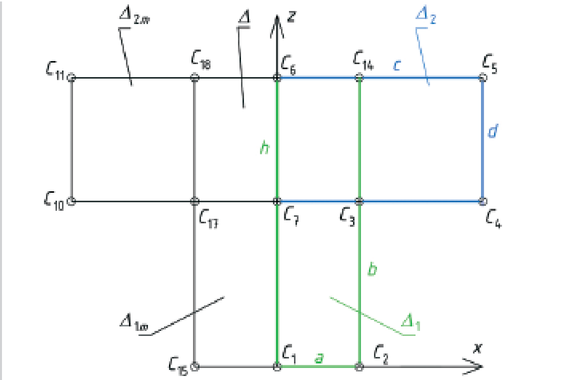

Figure 9.
Two pairs z-axis-symmetric auxiliary rectangles Δ_1, Δ_2 and Δ_{1m}, Δ_{1m} being the basis for determining the reference polygon and control composition

The longitudinal dimensions of the designed geometrical forms, including the roof and elevation ones, are prescribed at the second stage of the algorithm. Thus, a spatial reference figure called the control composition $\Gamma = <E_2F_2E_{15}F_{15}F_8F_{11}E_5E_{16}>$ is built at the present stage, Fig. 10.

On the basis of the reference polygon Δ, a next specific reference polygon P_r is created. Properties of $P_r = <C_2C_8C_5C_9C_{11}C_{16}C_{15}>$ will be considered in the next sections. Straight lines perpendicular to the plane of Δ passing through all vertices of the reference polygon P_r are passed. Some of the vertices of the control composition Γ are measured in these lines in arbitrary distances, but other in the distances depending on the adopted values of some new parameters. In the examples presented below each control composition Γ is regarded as a sum of four elevation tetragons, one rectangular base and one roof sector of a warped surface. Thus, relevant subsets of the edges of the control composition form closed borders modelling individual plane elevation walls and shell roof of the free form, see Fig. 10.

In the examples investigated in the present paper, all control compositions and architectural forms are axis-symmetrical. Such an assumption significantly simplifies further considerations and enables to obtain very attractive basic free forms.

The examples presented in next Sections 5 and 6 show, how a change of some proportions between parameters describing dimensions of selected roof or elevation elements affects the attractiveness and diversification of the designed architectural forms. Therefore an analysis of various sets of values assigned to the initially adopted parameters defining

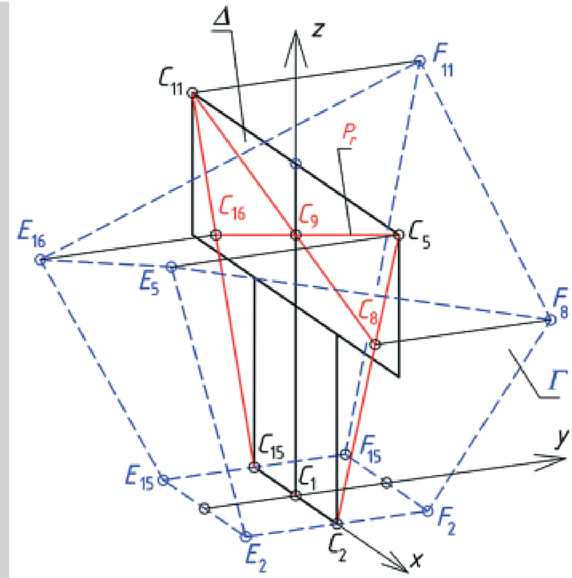

Figure 10.
Determination of eight vertices of control composition Γ

the designed geometrical forms in terms of their attractiveness and shape integration is conducted. For this purpose, various configurations of the above reference polygon P_r and control composition Γ are investigated.

The third step of the method's algorithm helps the designer to improve the process of shaping really innovative and attractive architectural forms due to adding a few parameters defining curved roof directrices. The method proposes to control some proportions between the ridges of these directrices and selected roof and wall elements. However, the positions of the roof directrices are limited so that they should be lain on elevation planes of the designed building form.

5. PRINCIPLES OF THE METHOD

5.1. Reference polygons

Properties of reference polygons were initially presented in the previous section, see Figs. 9, 10. A z-axis-symmetrical plane reference polygon Δ being a sum of four rectangles: a) two rectangles $\Delta_1 = <C_1C_2C_{14}C_6>$ and $\Delta_2 = <C_4C_5C_6C_7>$, b) other two rectangles $<C_1C_6C_{18}C_{15}>$ and $<C_7C_{10}C_{11}C_6>$ symmetrical towards z is presented in Fig. 11. For the edges of the above rectangles, the following parameters expressing their lengths are adopted: a) a and $h = b + d$ for Δ_1, b) c and d for Δ_2. The relevant vertices of these rectangles prescribe the reference polygon P_r = $<C_{15}C_2> \cup <C_2C_8> \cup <C_8C_5> \cup <C_5C_9> \cup <C_9C_8> \cup <C_{11}C_9> \cup <C_9C_{16}> \cup <C_{11}C_{16}> \cup <C_{16}C_{15}>$. Here, P_r defines a general transverse shape of the designed form.

Vertices of a spatial control composition Γ are determined on the basis of all vertices of P_r. Since Γ is formed with a tetrad of planes γ_i (i = 1 to 4), see Fig. 6, the point C_8 here has to be located on the segment C_2C_5. Therefore it is constructed as the point of the intersection of the segments C_2C_5 and C_4C_{11}. Similarly, the point C_{16} has to be the point of the intersection of the segments C_5C_{10} and $C_{11}C_{15}$.

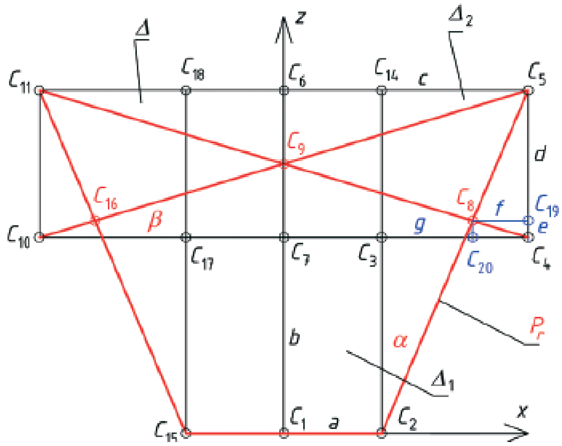

Figure 11.
First configurations of the reference polygons Δ and P_r

It is obvious that general shapes of such control compositions Γ and, in consequence, respective free forms depend on adopted proportions between the above parameters a, b, c and d. The figures Δ and P_r presented previously in Fig. 11 correspond to the following proportions: $b = 2a$, $c = 2.5a$ and $d = 1.5a$. An influence of these proportions on attractiveness of the shapes of the possible innovative building forms is considered in the present and next subsections. Another configurations of Δ and P_r related to the proportions $b = 0.5a$, $c = 0.75a$ and $d = a$ are shown in Fig. 12.

The above different configurations of the reference polygons Δ and P_r are used to create various attractive innovative free forms presented in Section 6.

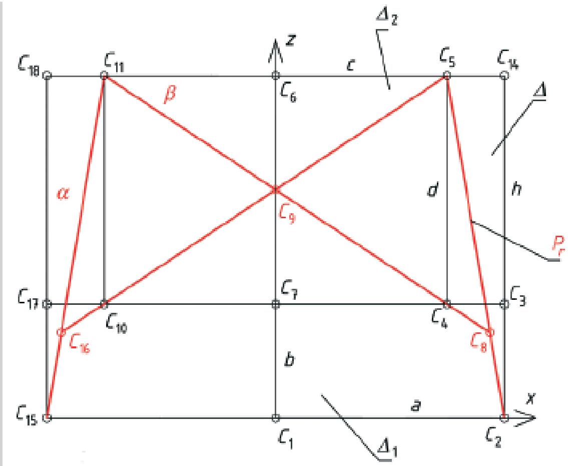

Figure 12.
Second configurations of the reference polygons Δ and P_r

5.2. Control compositions

Creation of control compositions on the basis of the polygons P_r and Δ consists in determining straight lines perpendicular to the plane containing the both polygons and passing through the vertices: C_1, C_2, C_5, C_8, C_9, C_{11}, C_{15}, C_{16} of P_r, see Fig. 13. The vertices: E_1, E_2, E_5, E_9, E_{15}, E_{16}, F_1, F_2, F_8, F_9, F_{11} and F_{15} of Γ are the points of the intersection of the straight lines and two planes $\gamma_1(E_2E_{15}E_5)$ and $\gamma_2(F_2F_{15}F_5)$ containing two faces of Γ.

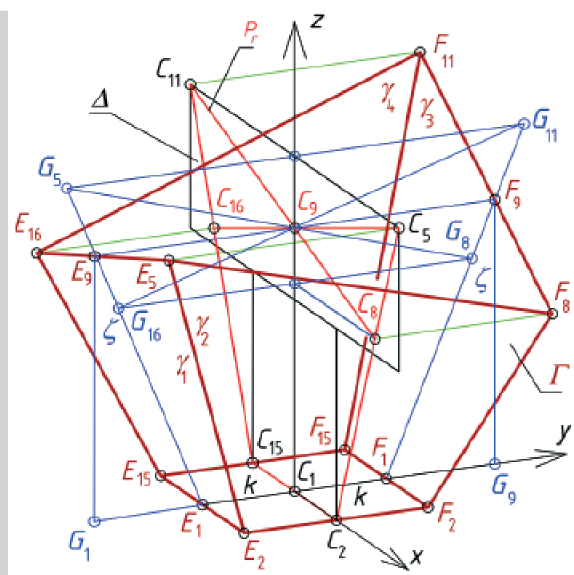

Figure 13.
On creating control compositions with reference polygons

For creating the control composition, at least two additional variables have to be adopted. There are: k defining the distance between the points C_1 and E_1 as well as ζ prescribing the angle between the plane γ_1 or γ_2 and the principal plane (x, z). Thus, the plane γ_1 is usually adopted as passing through the point E_1 belonging to y and inclined to the plane (x, z) at the angle ζ, where:

a) the point E_1 is located in the distance k from the point C_1,

b) the angle ζ can be measured between the straight line (E_1, E_9) and the z-axis.

The plane γ_3 can be found in the same way as γ_1. Here, γ_1 and γ_3 are (x, z)-plane-symmetrical. In addition, the axis z of the local co-ordinate system $[x, y, z]$ of Γ is the symmetry axis of Δ, Γ and P_r.

It is worth stressing that the points E_2, F_2, F_8, C_8 and E_5 are co-planar due to the fact that C_8 belongs to the segment C_2C_5, so the tetragon $<E_2F_2F_8E_5> \subset \gamma_2$. In an analogous manner the tetragon $<E_{15}F_{15}F_{11}E_{16}> \subset \gamma_4$ of Γ.

Summarizing, the above tetragons: $<E_2F_2F_8E_5>$ and $<E_{15}F_{15}F_{11}E_{16}>$ model two opposite elevation walls of a free form. Two other ones are shaped with the help of the polygons $<E_2E_{15}E_{16}E_5>$ and $<F_2F_{15}F_{11}F_8>$. The spatial quadrangle $B = <E_5F_8F_{11}E_{16}>$ is the border line of a shell roofing the designed building.

Three orthogonal projections of a control composition Γ_1 onto three principal projection planes and an axonometric projection are shown in Fig. 14. All faces of Γ_1 are transparent. The basic proportions of the composition Γ_1 are as follows: $b = 2a$, $c = 2.5a$, $d = a$, $k = a$, $\mathrm{tg}(\zeta) = \mathrm{tg}(\alpha) = 1/3$, where $a = 5000.0$ mm was assumed for the considered case.

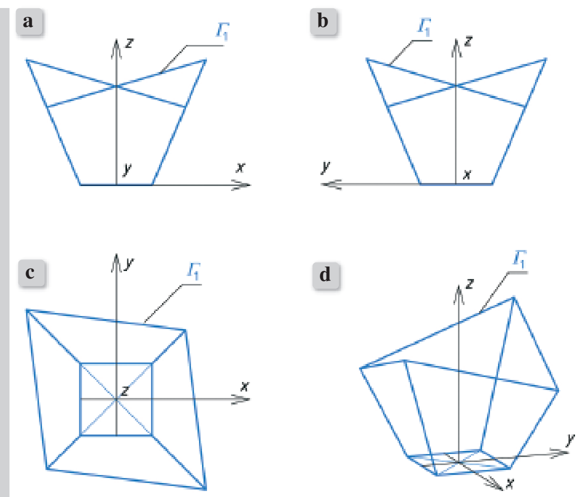

Figure 14.
a)-c) Three orthogonal projections onto three principal projection planes, d) an axonometric projection of a control composition Γ_1 of the first type

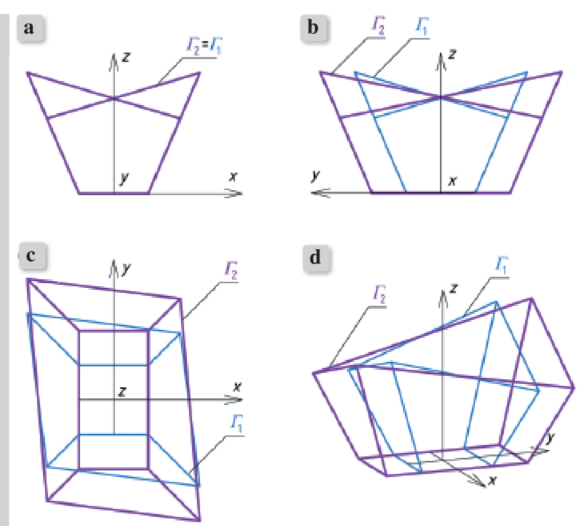

Figure 15.
a)-c) Three orthogonal projections onto three principal projection planes, d) an axonometric projections of two various control compositions Γ_1 and Γ_2 based on the same reference polygon of the first type

5.3. Configurations of the control compositions

Various configurations of the control compositions can be prescribed on the basis of the same arbitrary reference polygon P_r depending on values of the variables k and ζ. Two various ones Γ_1 and Γ_2 based on the same P_r are presented in Fig. 15. The considered Γ_1 was also used in the previous example.

The control composition Γ_2 differs from Γ_1 with of $k = 2a$. This new proportion produces a significant change in the longitudinal elevation shape of the corresponding free form.

To describe the really great possibilities related to shaping diversified configurations of the building free forms, two examples proposing a modification of some proportions between the examined parameters a, b, c, d, k and ζ are presented bellow. In the first example, the configurations of the reference polygons Δ and P_r, known from Fig. 12, are exploited to obtain Γ_1 of the second type, see Fig. 16. Here, there are three orthogonal projections of Γ_1 onto three principal projection planes and an axonometric projection, where its faces are shown as transparent. The proportions of the presented composition Γ_1 are as follows: $b = 0.5a$, $c = 0.75a$, $d = a$, $k = 2a$, $\mathrm{tg}(\zeta) = \mathrm{tg}(\alpha) = 1/4$. It was assumed that $a = 5000.0$ mm when creating Γ_1.

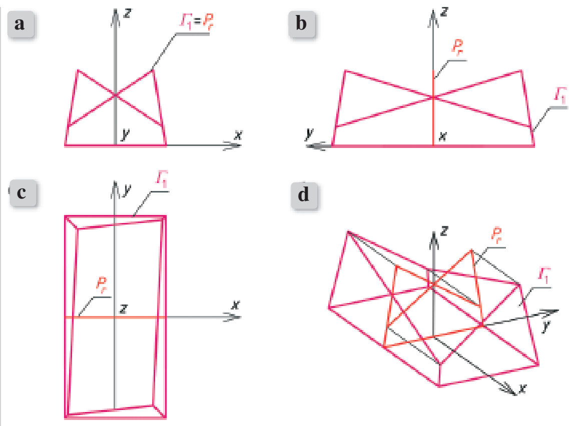

Figure 16.
a)-c) Three orthogonal projections onto three principal projection planes, d) an axonometric projection of the control composition Γ_1 based on the reference polygon of the second type

In the above example, the control composition Γ_1 becomes narrow in the direction compatible with the positive sense of the z-axis, see Fig. 16a, b. The both projections, Fig. 16a, b, give the possibility of comparing the influence of the proportion change between k and a from 1 to 2 if the observation directions are compatible with the directions of the axes x or y.

To observe the influence of various types of the reference polygon P_r on a control composition shape, three various sets of proportions of the considered parameters are assumed and three different control compositions corresponding to these proportions are created, see Fig. 17. For Γ_1, Γ_2 and Γ_3, the same value of a is adopted. The control composition Γ_1 was used in the previous example. The compositions Γ_2 and Γ_3 have the same values of $k = a$, $c = 2a$ and $d = 0.5a$. They differ from each other to the values of: a) the parameter b, which is equal to $0.5a$ for Γ_2 and a for Γ_3, b) the parameter ζ so that $\mathrm{tg}(\zeta) = 2/3$ for Γ_2 and $\mathrm{tg}(\zeta) = 1$ for Γ_3.

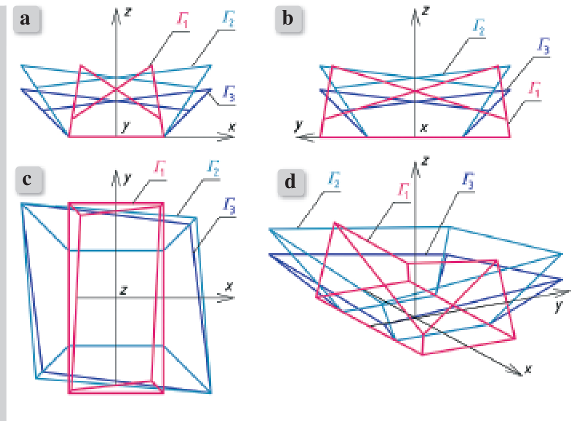

Figure 17.
a)-c) Three orthogonal projections onto three principal projection planes, d) an axonometric projections of three control compositions Γ_1, Γ_2 and Γ_3 based on two types of the reference polygons

5.4. Parametric shaping free forms

In the present section, an influence of a change of proportions between the parameters a, b, c and d on the attractiveness of the free form shapes is assumed for the case $c > a$. Analogous investigations, for the case $c < a$, are more complicated and require extended analysis because such forms become narrowing in the direction compatible with the positive values of the z-axis. This feature can produce troubles related to big twist degree of a roof shell. Problems corresponding to the curvature and overall dimensions of the roof shells and entire free forms are going to be discussed in another author's work. A comparison of results in terms of $a = c$ also goes beyond the scope of the present paper. Some initial results are presented in [7, 10, 12].

In the example below, three proportions between a and b were chosen arbitrarily. They are as follows: a) $b = 0.5a$, b) $b = a$, and c) $b = 2a$.

In the first case, the proportion $b = 0.5a$ is presented at the bottom right corner of the diagram shown in Fig. 18. Here, three various inclinations of elevation wall are distinguished with the help of three thick lines e_1, e_2 and e_3. Each of these lines corresponds to the relevant proportion: $c = 1.5a$, $c = 2a$ and $c = 2.5a$. In this case, the proportion $d = 0.5a$ is chosen as important. However, $d = a$ can be neglected because of too big proportion between the areas of the roof and corresponding elevation wall. If the inclination of an elevation wall to the vertical is small-

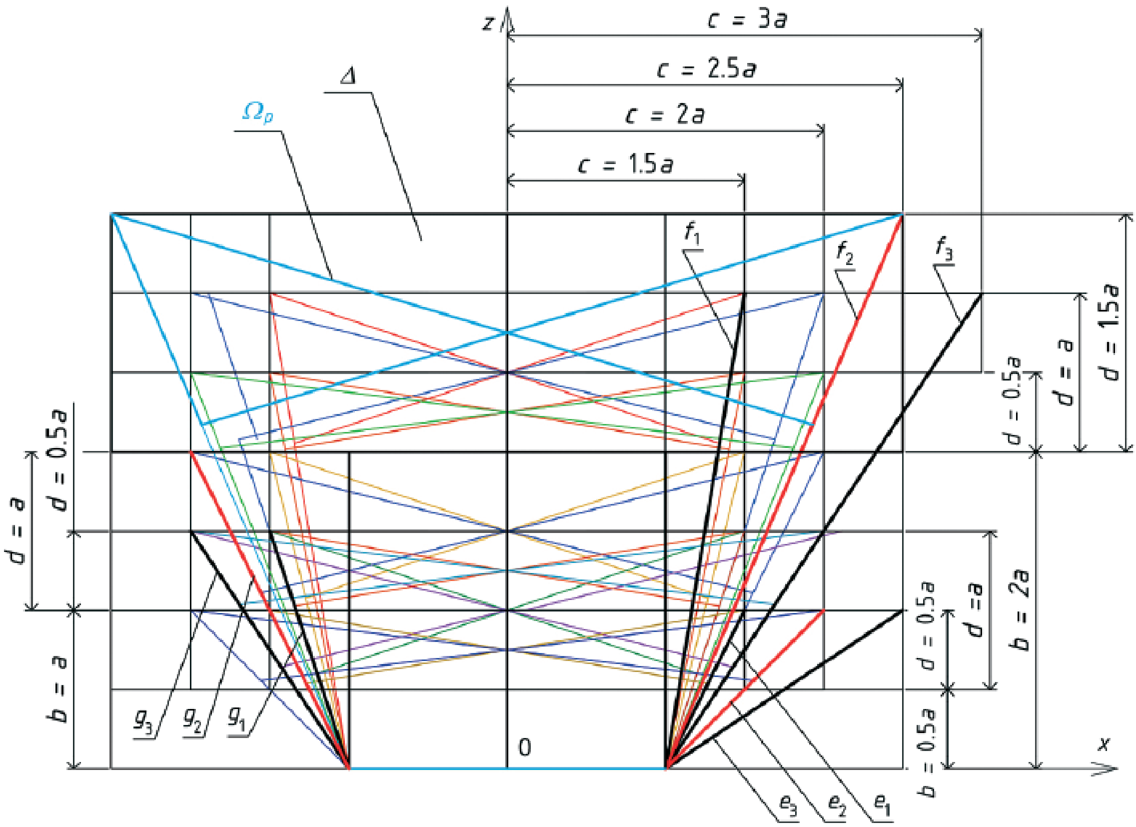

Figure 18.
Diagram of basic proportions between the parameters *a*, *b*, *c* and *d* expressing the border and optimal inclinations of elevation walls of various fine free forms to the vertical

er than the inclination of e_1 to the axis z, the influence of this inclination on the improvement of the attractiveness of an entire free form building in relation to a relevant free form with vertical elevation walls seems to be small. Therefore the line e_1 was chosen as the first border line. The second border line was adopted as e_3 because it seems to be very likely that every inclination of the elevation wall to the vertical bigger than the inclination of this line to the axis z can deteriorate the user's positive impression. As a result of the author's analysis, the line e_2 was chosen to express the optimal proportion enabling to obtain comfortable user's feeling and, next, an attractive geometrical building form.

For the second case $b = a$, it seems to be very reasonable to assume that the lines g_1 and g_3 are the border lines, and the line g_2 is close to the optimal one. Thus, in the author's opinion, the inclination of the line g_2 to the axis z corresponds to the very attractive inclination of elevation walls and attractive architectural shapes.

In the third case $b = 2a$, it seems to be reasonable to assume that the lines f_1 and f_3 are the border lines, and the line f_2 is close to the optimal one corresponding to the attractive inclination of these walls and attractive spatial shape.

The above border lines allow to obtain fine proportions between the considered parameters that are very close to the optimum ones leading to really attractive forms. In the author's opinion, fine proportions between the above mentioned parameters rely on the fact that dimensions of chosen elements or areas of a roof and relevant elevation wall will give a good impression of user and observer. Therefore, a roof cannot be to heavy in relation to the area and high of an elevation wall. Similarly, the inclination of the elevation should not evoke a negative emotion that the wall falls down.

If $b < 0.5a$, the horizontal spans of elevations in two orthogonal directions are relatively big and the influence of the curved roof on the observer's impression may be relatively small. A proportion between the areas of a shell roof and a proper elevation wall ensuring a good feeling of the user can be difficult to find. However, if $b > 2a$, the shell roof area can be too large to obtain a fine proportion and integration with the area of the proper elevation wall. In addition, in this case, the twist degree of the shell roof may be relatively big and such geometrical forms correspond to high-rise buildings.

For simplifying calculations leading to fine proportions of various attractive forms, a proper diagram was created, see Fig. 19. The points P_1, P_2 and P_3 distinguished on this diagram correspond to three lines e_2, f_2 and g_2 stressed in red in Fig. 18, for various proportions between b to a and c to a. The point P_2 is located near to the straight line $p(P_1, P_2)$ which represents proportions required by the author to achieve attractive shapes. Therefore the fine proportions between a, b and c can be calculated by means of the line p. However, many satisfactory proportions may be found with the help of the parallelogram limited by four lines including p_p and p_k parallel to p. These lines p_p and p_k are the result of the displacement of the section $P_1 P_2$ in positive and negative directions of the axis c in the distance $0.25a$. In addition, in the author's opinion, the variable d should be chosen from the range $<b, 1.5b>$. However, more accurate analyses are advisable.

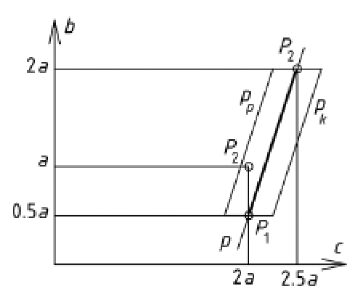

Figure 19.
Parallelogram limited by the lines p_p and p_k expressing fine proportions leading to attractive free forms

5.5. Shaping roof directrices for free forms

Parametric shaping of a roof shell requires defining two roof directrices at least [22, 23]. In the present paper, the process is reduced to one dependent variable that is a parameter w expressing the same length for the both ridges of these directrices.

In order to obtain the above aim the following assumptions are taken. The both directrices e and f are congruent circle arcs contained in two planes of opposite sides of control composition. The arcs e and f are built on the basis of two congruent circle arcs e_r and f_r contained in the plane of the reference polygon P_r, see Figs. 20 and 21d.

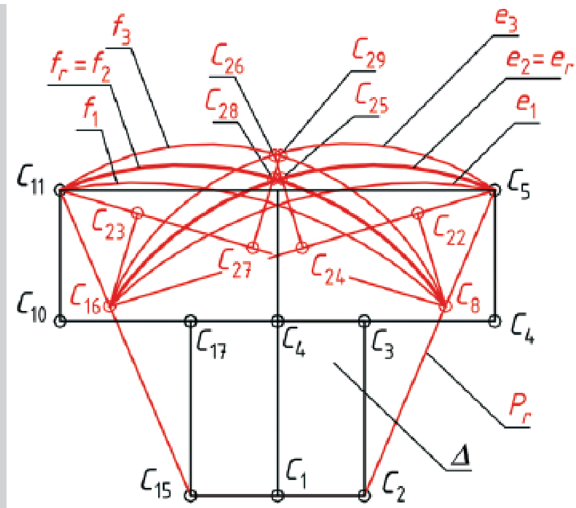

Figure 20.
Fine ridge length for roof directrix

Thus, the curve e_r was chosen from three auxiliary configurations e_1, e_2 and e_3 of circle arcs so that a fine proportion between its ridge $w = C_{24}C_{25}$ and the chosen edges of the polygon P_r would be achieved. Two ends of each of the above three arcs are adopted at the points C_5 and C_{16}, see Fig. 20. The ridges of the above arcs are measured along the bisectrix of the section C_5C_{16} from the point C_{24} in the distances equal to: $0.5C_8C_{22}$, $0.75C_8C_{22}$, C_8C_{22}. The point C_{22} is the intersection of the straight passing through the point C_8 and perpendicular to C_5C_{16}. It is obvious that C_{24} is the middle point of the chord C_5C_{16}.

Finally, the directrix e is constructed on the basis of $e_2 = e_r$ so that its ends are relevant vertices of Γ_1 and passes through the point N_{25}, see Fig. 21. Here, N_{25} is the point of the intersection of: a) the straight line n_{25} perpendicular to the plane of P_r and passing through the point C_{25}, b) the respective plane γ_1 of Γ_1 containing the above ends of e.

The directrix f_r can be created in an analogous way as e_r. Thus, its construction is based on the points C_{28}, N_{28} and the straight line n_{28} perpendicular to the plane of P_r. So the both directrices e and f are determined in the opposite planes γ_1 and γ_3 of the control composition, Fig. 21. An analysis of an influence of

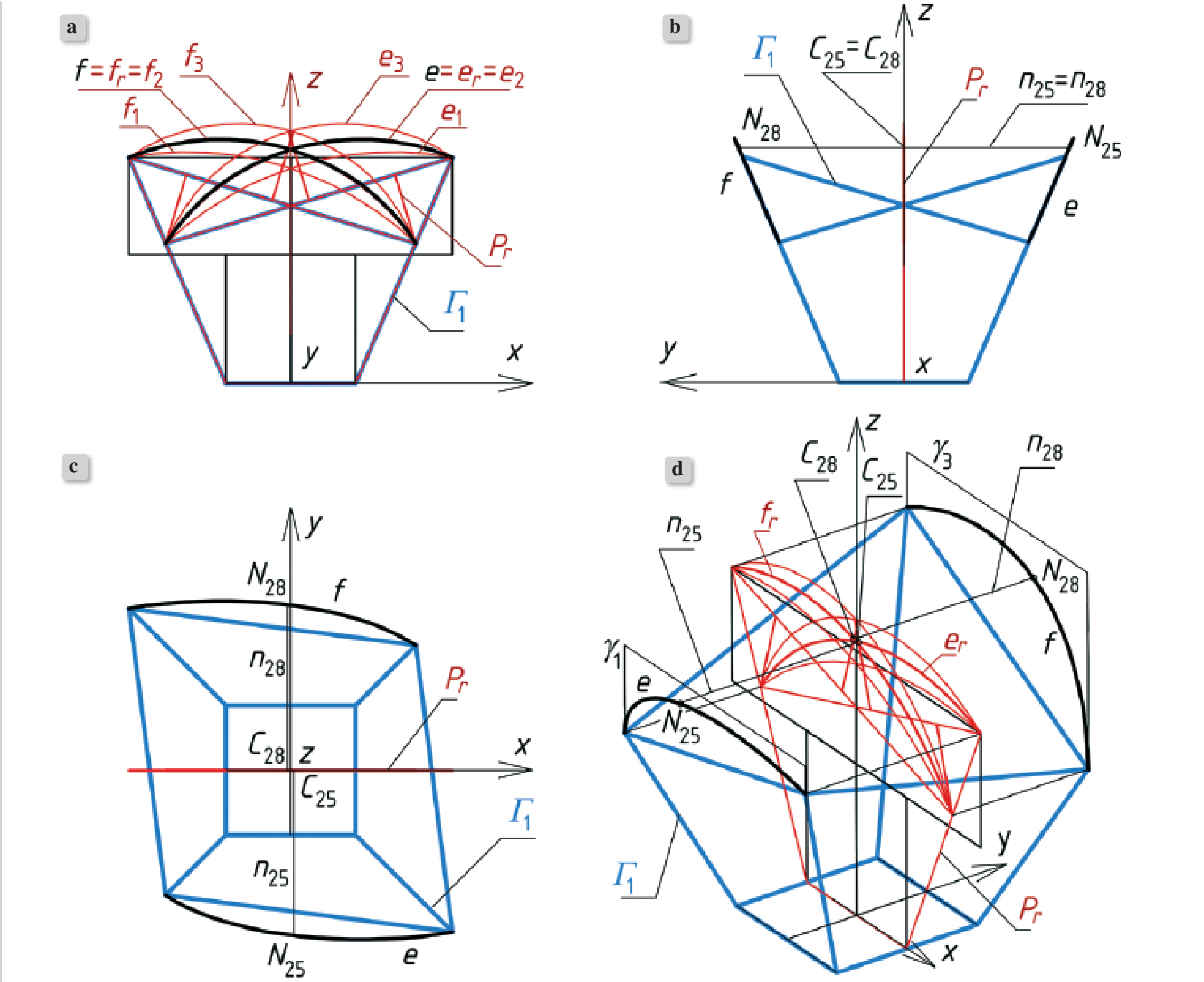

Figure 21.
Creation of arc roof directrices *e* and *f* contained in two opposite planes γ_1 and γ_3 of a control composition Γ_1; a)-c) Three orthogonal projections onto three principal projection planes, and d) an axonometric projection of Γ_1 based on the reference polygon and adopted auxiliary circle arcs e_r and f_r

the proportion between the parameters *w* and *a* is advisable, but it goes beyond the scope of the present paper.

6. ATTRACTIVE ARCHITECTURAL FREE FORMS

The analyses conducted in the previous sections enable the author to create two various interesting configurations of the innovative architectural free forms. He selected the fine proportions, with the help of the diagram presented in Fig. 19, for the first type of control compositions and the type of reference polyhedron P_r shown in Fig. 11. These proportions are as follows: $b = 2a$, $c = 2.5a$, $d = 1.5a$, $k = a$, $\mathrm{tg}(\zeta) = \mathrm{tg}(\alpha) = 1/3$, $w = 0.83a$. The basic variable *a* is equal to 5000.0 mm. A visualization of the free form built on the basis of the above proportions is shown in Fig. 22.

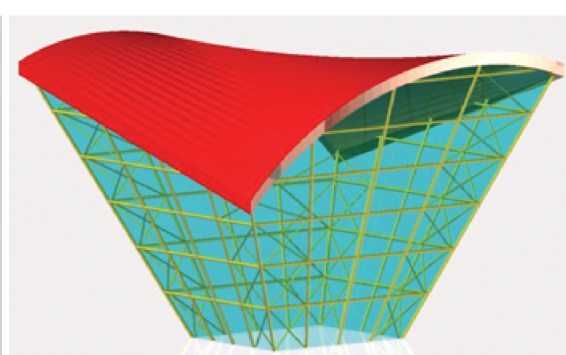

Figure 22.
Visualization of an attractive free form of the first type

The author also adopted the interesting proportions, as he hopes, for the second type of the control compositions and the type of reference polyhedron P_r presented in Fig. 12. These proportions are as follows: $c = 0.25a$, $b = 0.815a$, $d = 0935a$, so that $h = b + d = 1.75a$, $k = a$, $tg(\zeta) = tg(\alpha) = 1$, $w = 0.17a$. The basic variable a is equal to 10000.0 mm. A visualization of the respective free form built on the basis of the above proportions is shown in Fig. 23.

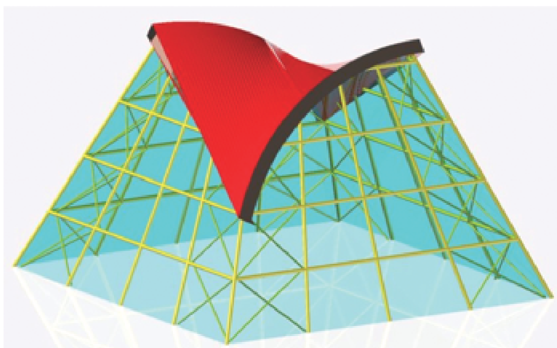

Figure 23.
Visualization of an attractive free form of the second type

The utilized proportions between variables *a, b, c, d*, *ζ*, *k* and *w* concern the general shapes of the chosen free forms. However, they are not only factors influencing on the attractiveness of these forms. To decide, which free form can be regarded as a satisfactory result, a few additional parameters should be investigated [21, 24]. Some of them should be related to: a) the location of the user, b) the direction of the user's observation, c) the built and natural environments, d) the function of the designed building forms, e) a type of materials. Some results of the author's initial observations including propositions for fine integration of the designed buildings with the built and natural environments are presented in the works [3, 9, 20]. Propositions of structural systems for the considered free forms can be found in [20, 25, 26]. Some author's information related to such shell buildings with corrugated steel roofs and their structures are little by little published on the website [27].

7. CONCLUSIONS

The proposed parametric description of the investigated innovative building free forms covered with corrugated shell steel roofs and oblique plane-walled glass elevations enables to create diversified and attractive configurations of their models in a very convenient way. It allows to use computer programs in effective seeking the free forms. The main advantage of the proposed method is to exploit as small variables as possible. Therefore only two types of rectangles are used to obtain the transverse characteristic of each considered free form. Usage of the geometrical parameters by the proposed method and computer programs creates great possibilities in seeking for really diversified and innovative shapes of the shell roof and oblique elevations. Thus, only basic forms characterized by axial symmetry are presented. Their derivative forms are going to be investigated in the next author's works.

Similarly, these innovative forms can easily be adapted to the existing built and natural environments. The number of the parameters and their proportions exploited by the method can be extended, in reasonable manner – even two times in a general case, to develop diversification of the checked free form configurations. Thus, the analyses conducted in the present paper should be extended to obtain more accurate proportions and another types of the attractive free form shapes. In addition, their complex structures should be analysed.

In the author's opinion, the really great possibilities of the proposed method in creating the diversified forms of buildings let the designer achieve big comfort for user or observer depending on the assumed point of the observation in three-dimensional space [21]. Therefore the method requires further development to take into account another aspects of shaping attractive architectural forms. For example, parameters related to location of these forms in the built and natural environments or possible directions of their observations should be taken into account.

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