# RELIABILITY ANALYSIS OF RIGID PILE CAPS USING AN ITERATIVE STRUT-AND-TIE MODEL

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Abstract

**The aim of this work is to present a new strut-and-tie model for design of rigid pile caps based on the concept of magnified area under the column. In this magnified area, compressive stresses have been reduced enough not to cause crushing of the struts. An iterative algorithm is used to determine the required depth of the magnified area. The model considers both failure by crushing of the compressed struts and by yielding of the tie reinforcement. A large number of experimental results available in the literature is used to test the model. The partial safety factors method is employed for pile caps design and structural safety is evaluated by means of the reliability index. The small failure probability, estimated through the reliability index, demonstrates the safety of the proposed method. A numerical example of practical use of the model is also presented.**

Streszczenie

**Celem pracy było przedstawienie nowego modelu S-T do projektowania sztywnych oczepów fundamentów palowych opartego na koncepcji zwiększonego pola u podstawy słupa. W obrębie tak powiększonego pola powierzchni naprężenia ściskające zostały zredukowane na tyle, aby nie doprowadzić do zmiażdżenia krzyżulców ściskanych. Algorytm iteracyjny został wykorzystany do określenia wymaganej głębokości odpowiadającej powiększonemu polu przekroju naprężeń. W modelu wzięto pod uwagę zarówno zniszczenie przez zmiażdżenie krzyżulców ściskanych, jak i w wyniku uplastycznienia prętów rozciąganych. W celu weryfikacji modelu wykorzystano dużą liczbę wyników badań literaturowych. Przy projektowaniu oczepów fundamentów palowych przyjęto częściowe współczynniki bezpieczeństwa, a bezpieczeństwo konstrukcji zostało określone przy przyjęciu wskaźnika niezawodności. Niskie prawdopodobieństwo zniszczenia, określone z wykorzystaniem wskaźnika niezawodności, pokazuje wysoki poziom bezpieczeństwa dla przedstawionej metody. Dodatkowo przedstawiono praktyczne zastosowanie modelu na przykładzie numerycznym.**

Keywords: **Strut-and-tie; Pile caps; Reinforced concrete; Structural design; Structural safety.**

## 1. INTRODUCTION

Usually, concrete structures are divided into two distinct regions, for which are made different assumptions about their behavior [1]. The regions in which the assumption of plane strain of the technical bending theory is good enough are called B-regions (B stands for beam and bending). For these regions the standard methods of the bending theory are applied. Standard methods are not applicable to all the other regions of a structure where the strain distribution is nonlinear, as near concentrated loads, corners, openings, deep beams, rigid pile caps, etc. These regions are called D-regions (D for discontinuity, disturbance or detail).

Thus, codes for reinforced concrete structures consider two different methods for design of pile caps. In the first method, the pile cap is analyzed as a beam or a slab supported on piles. The main reinforcement is calculated as in a bending problem, for the bending moment in a reference section located in the column. Shear strength is checked using the same criterion as in beams. Punching shear is verified as in slabs [2, 3, 4]. Usually, this sectional method is employed for flexible pile caps, where the distance between the axis of any pile to the column face is more than twice the height of the pile cap.

In order to avoid the necessity of one-way shear reinforcement, shear in a reference section is limited by the same formula used for thin slabs. The shear resistance depends on the compressive strength of concrete and reinforcement ratio [2, 3]. Some design codes [4, 5] also consider the slab thickness in the evaluation of the shear resistance. Usually, the reference section used to calculate the factored shear force is taken at a distance $d$ from the column face, where $d$ is the effective depth of the pile cap.

Failure by punching shear is checked in a control perimeter located at a distance $d/2$ from the column face [2, 6], or at a distance $2d$ [4, 5]. There is a lack of uniformity with respect to the location of the control perimeter as well as the value of punching shear resistance. Additional checks on the perimeter of the column cross section and around the piles may also be needed.

In the second method, pile caps are designed using a model of spatial truss, also called strut-and-tie model [7, 8, 9, 10]. The verifications aim to limit the compressive stresses in the concrete struts so as to prevent a brittle failure. If the struts are idealized as prismatic or uniformly tapered compression members [2], it is usually sufficient to limit the compressive stresses in the nodes of the truss, located near the piles and near the column. Then, the tie reinforcement is calculated. This method is employed for rigid pile caps, where the distance between the axis of any pile to the column face is less than twice the height of the pile cap [2, 4].

Nodal zones over the piles are referred to as CCT nodal zones because they receive two struts and one tie. Usually, an extended nodal zone is considered to take into account the dispersion of the contact stresses up to the level of reinforcement. Nodal zone under the column is referred to as CCC nodal zone because in a two-dimensional problem it receives three compressive forces. Several traditional strut-and-tie methods use an arbitrated value $x$ for the height of the CCC nodal zone under the column. In these methods, the value of $x$ is chosen without any rational criterion.

In a previous paper, the author [11] presented a strut-and-tie model for design of rigid pile caps based on the concept of magnified area under the column. In this magnified area, compressive stresses have been reduced enough not to cause crushing of the struts. Thus, this verification is replaced by determining the height $x$ of the nodal zone at the top of the pile cap (equal to the depth of the magnified area) required not to cause crushing of the struts. An iterative algorithm is used for this purpose.

The present paper introduces a modification in the geometry of the magnified area that simplifies the use of the model. The partial safety factors method is employed for pile caps design and structural safety is evaluated by means of the reliability index. A large number of experimental results available in the literature is used for determining the reliability index. The small failure probability, estimated through the reliability index, demonstrates the safety of the proposed method.

## 2. PROPOSED MODEL FOR DESIGN OF PILE CAPS

In the proposed model of this study, it is considered that the struts converge to a horizontal plane situated at a distance $x$ from the top of the pile cap. In this plane, the vertical stress $\sigma_{vd}$ has been reduced enough not to cause crushing of the struts. The compressive stress $\sigma_c$ in the strut near the top of the pile cap is given by $\sigma_c = \sigma_{vd} / \sin^2\theta$, where $\theta$ is the strut inclination. The inclination angle of the strut must satisfy the relationship $\tan\theta \geq 1/2$, in other words $\theta \geq 26.6°$. ACI Building Code [2] requires $\theta \geq 25°$. The height of the pile cap is chosen to ensure this minimum inclination for the concrete struts.

In order to avoid crushing of the struts near the top of the pile cap, it is necessary to limit $\sigma_c \leq f_{cd1}$, where $f_{cd1}$ is the design compressive strength of concrete in this zone. Therefore, the intended horizontal plane is one where $\sigma_{vd} \leq \sin^2\theta f_{cd1}$, as shown in Fig. 1.

As indicated in Fig. 1, the region with depth equal to $x$, located under the column, is nothing more than an extension of the column within the pile cap. In this region the column has an enlarged base. Since the column reinforcement penetrates to the bottom of the pile cap, or dowel bars are used, the design load is transferred progressively by adherence and mainly through the amplification of the compressed area inside the pile cap. Indeed, in the contact of the column with the top of the pile cap, only the load portion $N_{dc} = N_d - N_{ds}$ is transferred immediately to the

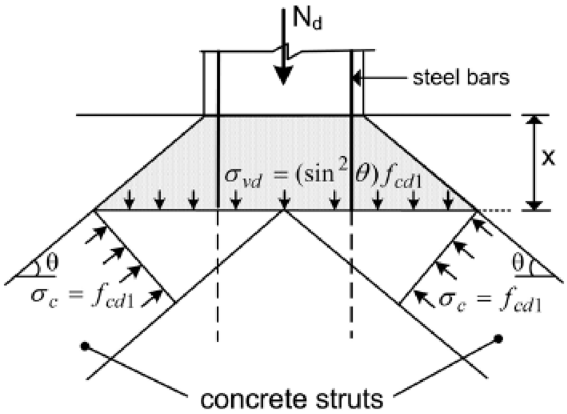


**Figure 1.**
**Verification of the concrete struts under the column**

concrete of the pile cap, where $N_{ds}$ is resisted by the column reinforcement. Failure due to bearing stress only occurs if the concrete of the pile cap has a much lower resistance than the concrete of the column.

Due to the confinement provided by the large concrete cover, concrete is subjected to a triaxial compression state (for pile caps on several piles), or a biaxial compression state (for two-pile caps). Thus, there is a significant increase in the uniaxial compressive strength $f_c$, with no risk of crushing in this zone even if the concrete of the pile cap has a strength somewhat lower than the concrete of the column.

Usually, this region is referred to as CCC nodal zone because in a two-dimensional problem it receives three compressive forces. Several design codes provide limits to the compressive stress in this nodal zone. For an unconfined node, the Eurocode EC2 [5] adopts $f_{cd1} = 1.0(1 - f_{ck}/250)f_{cd}$, where $f_{ck}$ is the characteristic strength in MPa and $f_{cd}$ is the uniaxial design compressive strength of concrete. According to this equation, it is found that $f_{cd1} \geq 0.85 f_{cd}$ if $f_{ck} \leq 37.5$ MPa. The limit $0.85 f_{cd}$ for the compressive stress is also adopted by [2, 3, 12]. So, in this work it is adopted $f_{cd1} = 0.85 f_{cd}$ for the CCC nodal zone.

In the previous paper [11], the magnified area under the column was defined considering a load distribution along the height of the pile-cap with the same inclination of the struts. In this work, the load distribution is considered as shown in Fig. 2 for a four-pile cap.

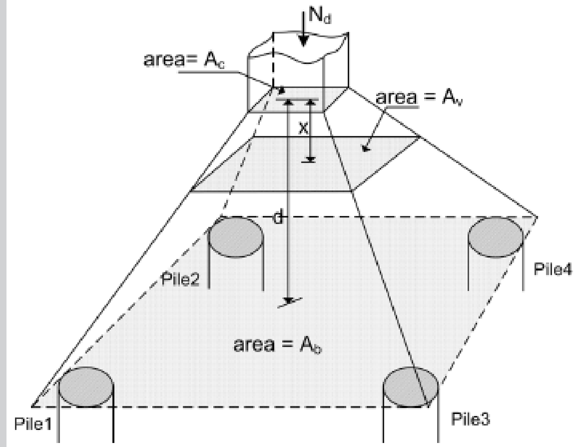


**Figure 2.**
**Load distribution in the vertical direction for a four-pile cap**

The area of the column cross section is $A_c$. The area of the base of the pile cap, in the outer contour of the piles, is $A_b$. The magnified area $A_v$ at depth $x$ is interpolated as

$$A_v = (1 - \xi) A_c + \xi A_b \tag{1}$$

where $\xi = x/d$ and $d$ is the effective depth of the pile cap.

Fig. 3 shows the area $A_b$ for two-pile and three-pile caps. Fig. 4 shows the proposed strut-and-tie model.

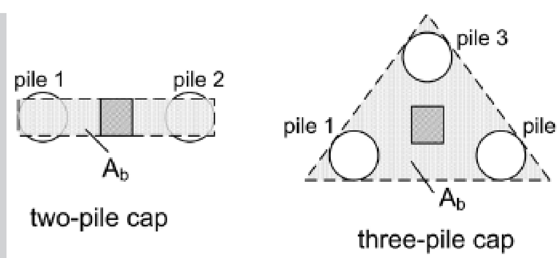


**Figure 3.**
**Area $A_b$ at the base of the pile caps**

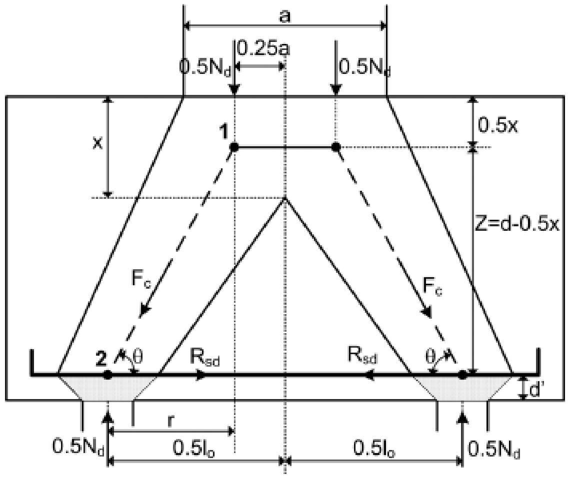


**Figure 4.**
**Proposed strut-and-tie model for two-pile caps**

The inclination of the struts is given by $\tan\theta = (d - 0.5x)/r$, where $r = 0.5l_o - 0.25a$. For pile caps on more than two piles, $r$ is the horizontal projection of the strut coming out of a point within the column and goes to the farthest pile. This equation can be written as

$$\tan\theta = \tan\theta_o\left(1 - 0.5\frac{x}{d}\right) \qquad (2)$$

where $\tan\theta_o = d/r$.

Thus, for a given value of $x$ it is possible to calculate the inclination of the compressive struts through the equation (2) and the magnified area $A_v$ under the column using equation (1). The vertical stress in the magnified area is given by $\sigma_{vd} = N_d/A_v$, where $N_d$ is the design load and $A_v$ is a function of $x$. Imposing the condition $\sigma_{vd} = \sin^2\theta f_{cd1}$, $x$ can be obtained through an iterative process.

Defining the relative normal force

$$\nu = \frac{N_d}{A_c\, f_{cd}} \qquad (3)$$

and imposing the condition $\sigma_{vd} = \sin^2\theta f_{cd1}$, where $f_{cd1} = 0.85\, f_{cd}$, results

$$\frac{\nu\, A_c f_{cd}}{(1-\xi)A_c + \xi A_b} = 0.85 f_{cd}\sin^2\theta \qquad (4)$$

Solving equation (4), $\xi$ is given by

$$\xi = \frac{x}{d} = \frac{\nu - 0.85\sin^2\theta}{(\eta - 1)0.85\sin^2\theta} \qquad (5)$$

where $\eta = A_b/A_c$.

Equation (5) can only be solved iteratively. For this purpose, the following procedure is adopted:

Step 1: Assume $x = 0$ and calculate the angle $\theta = \theta_o$ through the relationship $\tan\theta_o = d/r$.

Step 2: Compute $x = \xi d$ by means of the equation (5). If $x \leq 0$, the solution is $x = 0$, indicating that the struts can converge to the top of the pile cap. If $x > 0$, go to the next step.

Step 3: With the value of $x$ obtained in the previous step, compute a new angle $\theta$ by means of the equation (2). With this value $\theta$, calculate the new value of $x$ through the equation (5). Proceed iteratively until convergence of $x$. The adopted convergence criterion is: $|x_j - x_{j-1}|/x_j < 0.01$, where $x_{j-1}$ and $x_j$ are the values obtained in two successive iterations.

To ensure a minimum ductility and prevent brittle failure, the relative depth $\xi = x/d$ of the horizontal plane obtained from equation (5) should be limited. Thus, $\xi$ is restricted to the values $\xi \leq 0.45$ (for $f_{ck} \leq 35$ MPa) and $\xi \leq 0.35$ (for $f_{ck} > 35$ MPa), according to CEB-FIP Model Code [13] recommendations. Similarly, the angle of inclination of the struts is limited to $\theta \geq 26.6°$. If these restrictions are not met, the effective depth $d$ of the pile cap and/or the dimensions of the column cross section must be increased. The same should be done if the iterative process does not converge.

To avoid the iterative process, it may be adopted a minimum value for the angle $\theta$, computing $x$ by means of equation (5). In [14], for example, it is adopted $\theta = 26.6°$ as being this minimum strut inclination. This procedure simplifies the structural design but it can be uneconomical, particularly for two-pile caps. Other authors [15,16] consider a fixed value for $x$, such as $x = 0.30d$, and the strut inclination $\theta$ is obtained from the equation (2). With this simplification, we obtain $Z = 0.85d$. This value of $x$ can be excessive if the column is not heavily loaded, and the solution is uneconomical. Anyway, the proposed iterative process involves simple calculations as well as converges very quickly, being the recommended solution.

If the column transmits a bending moment to the pile cap, the relative normal force ν must be calculated for an equivalent load $N_{de} > N_d$ . As a simplification, one can consider that $N_{de}$ is equal to the number of piles multiplied by the design reaction of the most loaded pile.

Once $x$ is known, the lever arm $Z = d - 0.5x$ is defined as shown in Fig. 4. Figs. 5 and 6 show the variations of the relative lever arm $Z/d$ for two ratios $\eta = A_b/A_c$. It should be noted that the simplified value $Z = 0.85d$ is excessive in many cases. These Figs. can be used to determine the minimum effective depth of the pile cap. For example, for a pile cap with $\eta = 4$ and $\nu = 0.6$, it is necessary that $d/r > 0.75$ as can be seen in Fig. 5. If $d/r = 1.00$, results $Z/d = 0.90$. Similar graphs can be obtained for other ratios of $\eta = A_b/A_c$.

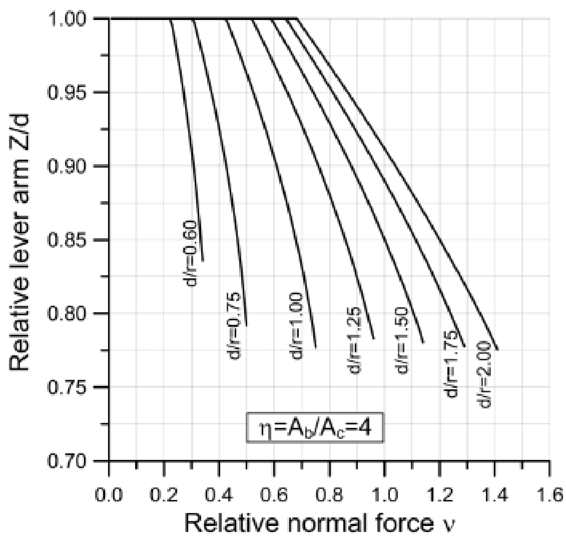


**Figure 5.**
**Relative lever arm $Z/d$ for $\eta = 4$**

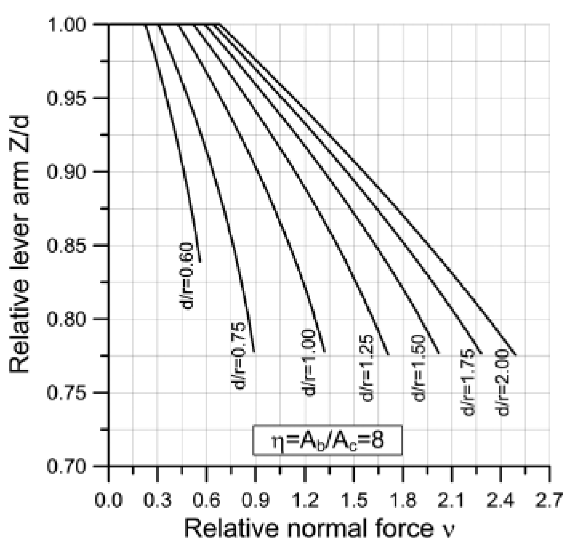


**Figure 6.**
**Relative lever arm $Z/d$ for $\eta = 8$**

Once the lever arm $Z$ is obtained, the tie steel area is calculated as $A_s = R_{sd}/f_{yd}$, where $R_{sd} = 0.5N_d \cot\theta$ and $\cot\theta = (0.5l_o - 0.25a)/Z$. Therefore,

$$A_s = \frac{0.5N_d\left(0.5l_o - 0.25a\right)}{Z f_{yd}} \quad (6)$$

where $f_{yd}$ is the design yield strength of the steel.

Equation (6) may be written as

$$A_s = \frac{M_d}{Z f_{yd}} \quad (7)$$

where $M_d$ is the design bending moment in a reference section located at a distance $0.25a$ behind the column face caused by the pile reaction.

The pile reactions are obtained considering the pile cap as a rigid body and each pile is modeled as a spring element. If the piles are loaded unequally due to the eccentricity of the force $N_d$, it must be considered the one that causes the largest value of $M_d$ in the reference section. The model may be used to calculate the reinforcement of pile caps supported on several piles. In order to do this, just calculate the reaction of each pile and determine the maximum bending moment in the reference section. This bending moment is calculated considering the reactions of all piles located at the same side of the analyzed section.

Fig. 7 indicates the sections for calculation of the reinforcement in two orthogonal directions, for a pile cap with many piles. The reinforcement in direction 1 is calculated for the bending moment in the section S1, caused by the reactions of all piles located on the right of this section. If the piles on the left cause a higher bending moment, one should consider the section S1 located in this side. The reinforcement in direction 2 is calculated for the bending moment in the section S2, in a similar way. The reinforcement in each direction should be concentrated in the alignment of the piles. Special attention should be given to anchoring of the tie bars. The minimum force to be anchored must be greater than or equal to 75 percent of the maximum tensile force [17]. Additional reinforcement placed between the piles may be required to control concrete cracking. If some piles are tensioned, a negative bending moment will appear which will require reinforcement in the upper face of the pile cap. Furthermore, it should be ensured that the steel bars of the tensioned piles are anchored at the top of the pile cap.

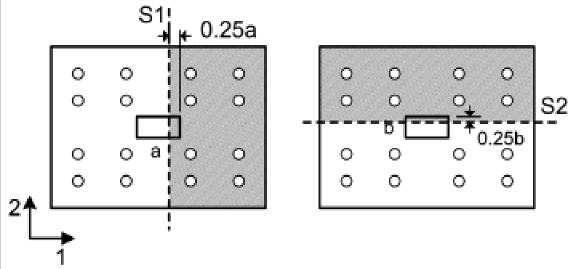


**Figure 7.**
**Reference sections for calculation of the reinforcement**

The strut stress at node 2 (Fig. 4) on the pile is given by

$$\sigma_{c2} = \frac{F_{de}}{A_{amp}\sin^2\theta} \tag{8}$$

where $F_{de}$ is the design pile reaction, $A_{amp} = kA_p$ is the amplified area on the pile, $A_p$ is the area of the pile cross section and $k$ is a factor that takes into account the spreading of the contact stresses to the centroid of the reinforcement.

For two-pile caps subjected to a centered load as shown in Fig. 4, $F_{de} = 0.5N_d$. If the piles are unequally loaded, one should determine the node where $\sigma_{c2}$ is maximum.

It is assumed that the contact stresses on the piles spread at 45 degree angles in all directions, according to the recommendations of EC2 [5]. Thus, the coefficient $k$ is given by

$$k = \left(1 + \frac{2d'}{\phi_p}\right)^2 \tag{9}$$

where $\phi_p$ is the diameter of the pile cross section and $d'$ is the distance between the reinforcement axis and the bottom face of the pile cap as shown in Fig. 4.

For piles of square section, $\phi_p$ is the side of the cross section. It is necessary to ensure that the amplified area $A_{amp} = kA_p$ do not fall out of the pile cap. In all cases, it is recommended to limit $k \le 4$.

For two-pile caps, the spreading occurs only in the direction of the piles and $k = 1 + 2d'/\phi_p$. If it is necessary to consider the bidirectional amplification given in equation (9) to avoid crushing of the strut, transverse reinforcement is required to restraint vertical splitting on the pile. This can be modeled using a transverse strut-and-tie model. The area of this transverse reinforcement is given by $A_{st} = 0.25F_{de}/f_{yd}$. This reinforcement is not necessary for pile caps with more than two piles.

The nodal zone over the pile is referred to as CCT nodal zone because it receives two struts and one tie. In order to avoid crushing of the struts it is necessary to limit $\sigma_{c2} \le f_{cd2}$. Here there is no consensus about the limit to the compressive stress $f_{cd2}$, as shown in [11]. So, conservatively, it is adopted

$$f_{cd2} = 0.60\left(1 - \frac{f_{ck}}{250}\right)f_{cd} \tag{10}$$

according to CEB-FIP Model Code [13] recommendations.

## 3. VERIFICATION OF THE STRUCTURAL SAFETY

Structural safety may be evaluated comparing its resistance to external loads. The difference between these two values is a measure of the distance to the ultimate limit state. Considering resistance and loads as random variables, it is necessary to formulate the problem in terms of the failure probability. If $F_u$ is a random variable representing the failure load of the structure (i.e., its load capacity), and $F_s$ represents the applied loads, the failure probability $p_F$ is given by

$$p_F = P(F_s > F_u) \tag{11}$$

and indicates the probability of external actions exceeding the structural resistance [18].

This problem can be formulated in terms of the safety margin or of the safety coefficient [19]. The second alternative is adopted in this work. Defining the safety coefficient $S = F_u/F_s$, failure corresponds to the occurrence of the event $S < 1$. If $S$ has a lognormal distribution, then its natural logarithm $Y = \ln S$ has a normal distribution with mean value $\mu_Y$ and standard deviation $\sigma_Y$. Then, the failure probability is given by

$$p_F = \int_{-\infty}^{0} f_Y(y)dy \tag{12}$$

where $f_Y(y)$ is the normal or Gaussian distribution.

Defining the standard reduced variable $t = (y - \mu_Y)/\sigma_Y$, equation (12) can be written as

$$p_F = \Phi(-\beta) = \int_{-\infty}^{-\beta} f_Y(t)\sigma_Y\, dt \tag{13}$$

where $\beta = \mu_Y/\sigma_Y$ is the reliability index and $\Phi(-\beta)$ is the cumulative function of the reduced normal distribution.

As can be observed, the failure probability reduces as $\beta$ increases. Thus, $\beta$ index is an important measure of the safety level once it is related to the failure probability or, alternately, to the structural reliability.

In order to demonstrate the validity of the proposed model, 186 pile caps tested by other authors have been analyzed. These experimental results include 37 two-pile caps, 21 three-pile caps and 128 four-pile caps. All pile caps were subjected to a centered load. The columns have square or rectangular cross section. The pile sections can be square, rectangular or circular. Concrete compressive strength $f_c$ based on tests of cylinders varies from 13.2 MPa to 49.3 MPa.

Table 1 shows summary information about the pile caps. Full details may be obtained in references listed in the table. For four-pile caps, the complete data may be obtained in [29].

**Table 1.**
**Pile caps used for checking the model**

| Ref. [ ] | Number of pile caps | Number of piles | $f_c$ MPa min | $f_c$ MPa max |
|---|---|---|---|---|
| [20] | 11 | 2 | 32.8 | 33.9 |
| [21] | 20 | 2 | 19.5 | 32.3 |
| [22] | 6 | 2 | 23.6 | 47.0 |
| [22] | 12 | 3 | 17.7 | 37.4 |
| [23] | 9 | 3 | 24.5 | 40.3 |
| [22] | 31 | 4 | 13.2 | 49.3 |
| [24] | 13 | 4 | 22.5 | 43.7 |
| [25] | 19 | 4 | 18.9 | 30.9 |
| [26] | 17 | 4 | 25.6 | 30.9 |
| [27] | 30 | 4 | 24.5 | 29.4 |
| [28] | 18 | 4 | 20.2 | 37.9 |

When carrying the structural design, the design load $N_d$ is given by $N_d = \gamma_f N_k$ , where $N_k$ is the characteristic load and $\gamma_f > 1$ is a partial safety factor. Design compressive strength of concrete is $f_{cd} = f_{ck}/\gamma_c$, where $f_{ck}$ is the characteristic strength and $\gamma_c > 1$ is other partial safety factor. Finally, design yield strength of reinforcement is $f_{yd} = f_{yk}/\gamma_s$, where $f_{yk}$ is the characteristic yield strength and $\gamma_s > 1$ is a third partial safety factor. For comparison with experimental results, it is adopted $f_{ck} = f_c$ and $f_{yk} = f_y$ , where $f_c$ and $f_y$ are the strengths obtained in the tests. The partial safety factors $\gamma_c = 1.50$ and $\gamma_s = 1.15$ are adopted in accordance with EC2 [5]. For the load factor, $\gamma_f = 1.40$ is assumed to be a mean value in accordance with EN 1990 [30].

The load capacity $F_u$ is the experimental failure load. The applied load $F_s$ is equal to the characteristic load $N_k = N_d/\gamma_f$. Then, the safety coefficient $S = F_u/F_s$ and $Y = \ln S$ for each particular test can be determined.

Considering all tests for each type of pile cap, one can determine the mean value $\mu_Y$ and the standard deviation $\sigma_Y$. Finally, the reliability index $\beta = \mu_Y/\sigma_Y$ and the failure probability $p_F = \Phi(-\beta)$ can be calculated.

To determine the theoretical failure load, an incremental process is employed for the load $N_d$. For each value of $N_d$, it is determined the depth $x$ of the horizontal plane with the iterative algorithm presented previously. Then the compressive stress in the strut on the pile with use of equations (8), (9) and (10) is verified. Finally, the tensile force $R_{sd}$ in the tie is compared with its strength $A_s f_{yd}$. If the rupture does not occur for any of these two failure modes, the load is increased to find $N_d$. The applied load $F_s$ to calculate the safety coefficient is $F_s = N_d/\gamma_f$ .

## 4. RESULTS AND DISCUSSION

Fig. 8 shows the normal probability paper for $Y = \ln S$ for 37 two-pile caps. In the same figure, the result of the Kolmogorov-Smirnov test (K-S test) is indicated for a fit with the normal distribution. For a 5% significance level and 37 sample points, the table for K-S test [31] provides $D_{37}^{0.05} = 0.22$. As $D < D_{37}^{0.05}$, it can be assumed that the safety coefficient $S$ is log-normal.

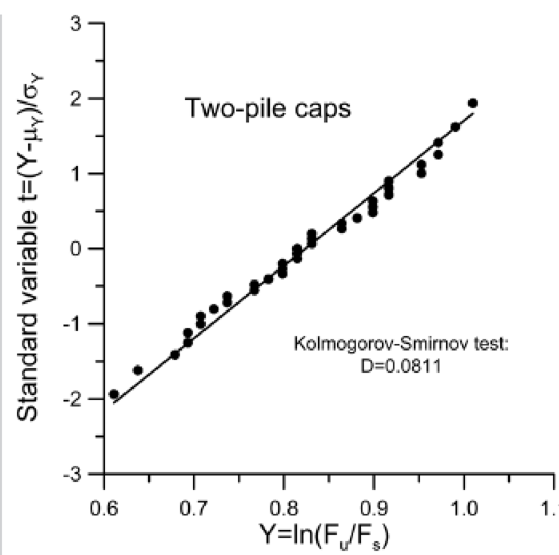


**Figure 8.**
**Probability paper for two-pile caps**

Fig. 9 shows the histogram of $Y = \ln S$ for 37 two-pile caps. The mean value of $Y$ is $\mu_Y = 0.82$ and the standard deviation is $\sigma_Y = 0.10$. The reliability index is $\beta = 8.2$.

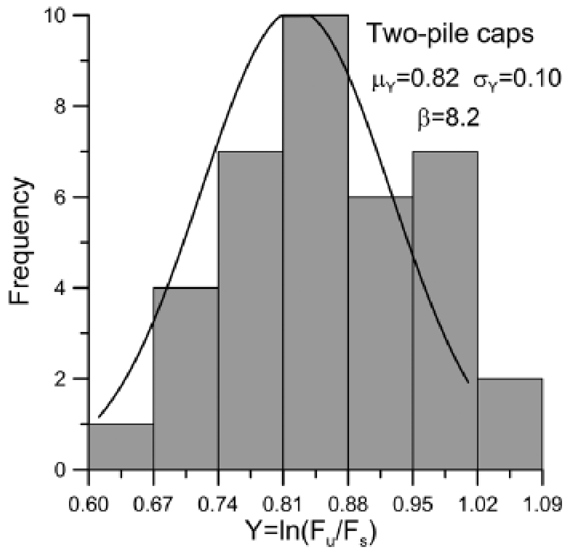


**Figure 9.**
**Histogram for two-pile caps**

Fig. 10 shows the normal probability paper for $Y = \ln S$ for 21 three-pile caps and the result of the K-S test. For a 5% significance level and 21 sample points, the table for K-S test provides $D_{21}^{0.05} = 0.29$. As $D < D_{21}^{0.05}$, it can be assumed that the safety coefficient $S$ is lognormal.

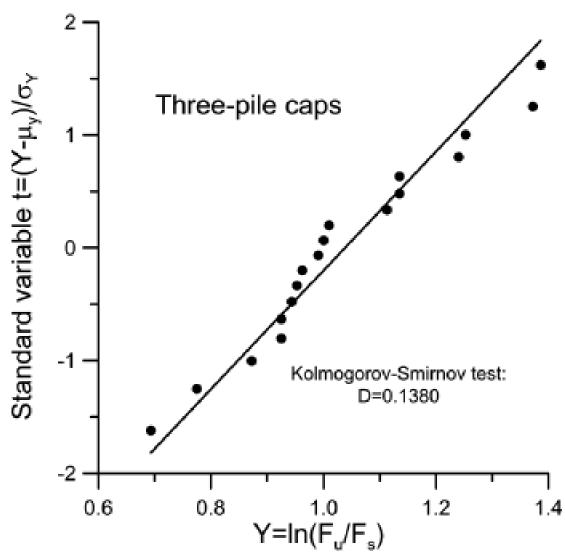


**Figure 10.**
**Probability paper for three-pile caps**

Fig. 11 shows the histogram of $Y = \ln S$ for 21 three-pile caps. The mean value is $\mu_Y = 1.04$, the standard deviation is $\sigma_Y = 0.18$ and the reliability index is $\beta = 5.8$.

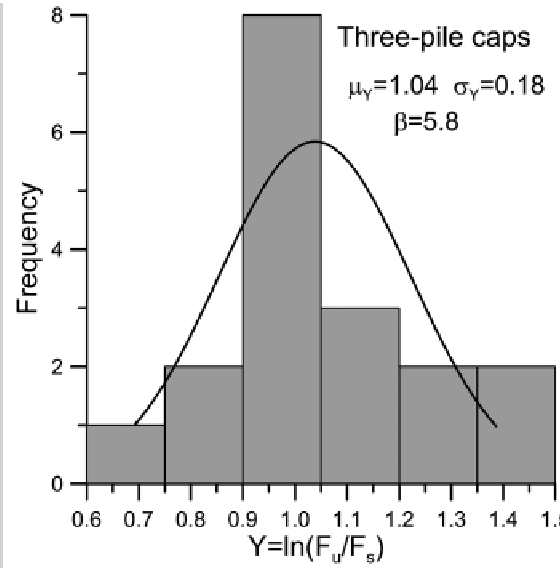


**Figure 11.**
**Histogram for three-pile caps**

Fig. 12 shows the normal probability paper for $Y = \ln S$ for 128 four-pile caps and the result of the K-S test. For a 5% significance level and 128 sample points, the table for K-S test provides $D_{128}^{0.05} = 0.12$. As $D < D_{128}^{0.05}$, it can be assumed that the safety coefficient $S$ is lognormal.

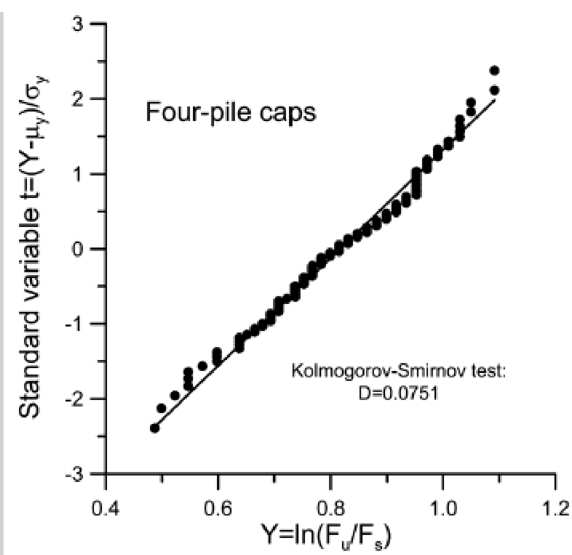


**Figure 12.**
**Probability paper for four-pile caps**

Fig. 13 shows the histogram of $Y = \ln S$ for 128 four-pile caps. The mean value is $\mu_Y = 0.82$, the standard deviation is $\sigma_Y = 0.14$ and the reliability index is $\beta = 5.9$.

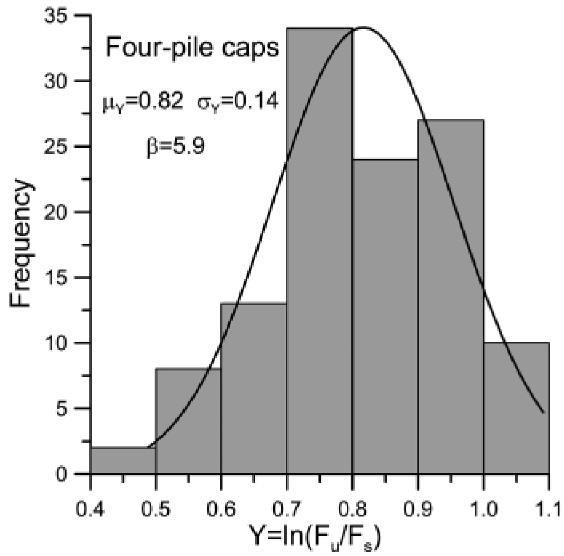


**Figure 13.**
**Histogram for four-pile caps**

As can be noted, the reliability index varies according to the number of piles. The highest value for the reliability index $\beta$ was obtained for two-pile caps. This high value for $\beta$ is due to the fact that the standard deviation $\sigma_Y$ is very small.

Table 2 shows a summary of the results, together with the failure probability $p_F = \Phi(-\beta)$.

**Table 2.**
**Reliability index and estimated failure probability**

| Number of piles | $\mu_Y$ | $\sigma_Y$ | $\beta = \frac{\mu_Y}{\sigma_Y}$ | $p_F$ |
|---|---|---|---|---|
| 2 | 0.82 | 0.10 | 8.2 | $0.12 \times 10^{-15}$ |
| 3 | 1.04 | 0.18 | 5.8 | $0.33 \times 10^{-8}$ |
| 4 | 0.82 | 0.14 | 5.9 | $0.18 \times 10^{-8}$ |

EN 1990 [30] adopts a target reliability index $\beta = 3.8$ for medium failure consequence in a reference period of 50 years. For high failure consequence the target reliability index is $\beta = 4.3$. For a reference period of 1 year, these target indexes rise to $\beta = 4.7$ and $\beta = 5.2$, respectively. These target reliability indices were achieved for all pile caps.

## 5. NUMERICAL EXAMPLE

The proposed model is used to calculate the two-pile cap shown in Fig. 14.

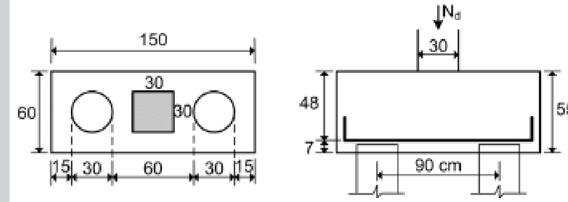


**Figure 14.**
**Example of two-pile cap**

Additional data:

$$N_d = 1600 \text{ kN}; \; f_{ck} = 30 \text{ MPa}; \; f_{yk} = 500 \text{ MPa}$$

Design strength of concrete:

$$f_{cd} = 30/1.5 = 20 \text{ MPa } (= 2 \text{ kN/cm}^2)$$

Design strength of reinforcement:

$$f_{yd} = 500/1.15 = 435 \text{ MPa } (= 43.5 \text{ kN/cm}^2)$$

Relative normal force:

$$\nu = \frac{N_d}{A_c f_{cd}} = \frac{1600}{30x30x2} = 0.89$$

$$r = 0.5 l_o - 0.25a = 0.5x90 - 0.25x30 = 37.5 \text{ cm}$$

$$\tan\theta_o = \frac{d}{r} = \frac{48}{37.5} = 1.28 \rightarrow \theta_o = 52^o$$

Area of the base (see Fig. 3):

$$A_b = 30(30 + 60 + 30) = 3600 \text{ cm}^2$$

$$\eta = \frac{A_b}{A_c} = \frac{3600}{30x30} = 4$$

Table 3 shows the results of the iterative process to find $x$, using equations (2) and (5).

**Table 3.**
**Results of the iterative process**

| Iteration | $x$ (cm) | $\theta$ (degrees) | $\frac{x_j - x_{j-1}}{x_j}$ | $\frac{x}{d}$ |
|---|---|---|---|---|
| 0 | 0 | 52.00 | --- | 0 |
| 1 | 10.98 | 48.58 | 1.00 | 0.23 |
| 2 | 13.79 | 47.62 | 0.20 | 0.29 |
| 3 | 14.70 | 47.31 | 0.06 | 0.31 |
| 4 | 15.01 | 47.20 | 0.02 | 0.31 |
| 5 | 15.12 | 47.16 | <0.01 | 0.32 |

Thus: $x = 15.12$ cm; $x/d = 0.32 < 0.45$;

$\theta = 47.16^o > 26.6^o$

Lever arm:

$$Z = d - 0.5x = 48 - 0.5x15.12 = 40.44 \text{ cm}$$

$$Z/d = 40.44/48 = 0.84$$

Employing the Fig. 5 with $d/r = 1.25$ and $\nu = 0.90$, $Z/d = 0.83$ is obtained without performing the iterations. The final result of the iterative process shown in Table 3 is practically the same as that obtained in [11]. The advantage of the present formulation is the possibility of drawing graphs as shown in Figs. 5 and 6, which allow the direct solution of the problem.

Tie steel area: $A_s = \dfrac{0.5x1600x37.5}{40.44x43.5} = 17.05 \text{ cm}^2$

Strut verification on the pile:

$$f_{cd2} = 0.60\left(1 - \frac{f_{ck}}{250}\right) f_{cd} = 10.56 \text{ MPa}$$

$$F_{de} = 0.5N_d = 800 \text{ kN (pile reaction)}$$

$$A_p = 707 \text{ cm}^2 \text{ (pile cross section area)}$$

$$k = 1 + \frac{2d'}{\phi_p} = 1 + \frac{2x7}{30} = 1.47$$

(considering unidirectional spreading)

$A_{amp} = kA_p = 1039 \text{ cm}^2$ (magnified area on the pile)

$$\sigma_{c2} = \frac{F_{de}}{A_{amp} \sin^2 \theta} = \frac{800}{1039 \sin^2 47.16} =$$

$$= 1.43 \text{ kN/cm}^2 \; (\sigma_{c2} = 14.3 \text{ MPa})$$

Since $\sigma_{c2} > f_{cd2}$, it is necessary to consider the bidirectional spreading.

$k = \left(1 + \dfrac{2d'}{\phi_p}\right)^2 = \left(1 + \dfrac{2x7}{30}\right)^2 = 2.15 < 4$ (considering bidirectional spreading)

$A_{amp} = kA_p = 1520 \text{ cm}^2$ (magnified area on the pile)

$$\sigma_{c2} = \frac{F_{de}}{A_{amp} \sin^2 \theta} = \frac{800}{1520 \sin^2 47.16} =$$

$$= 0.98 \text{ kN/cm}^2 \; (\sigma_{c2} = 9.8 \text{ MPa})$$

Since $\sigma_{c2} < f_{cd2}$, the strut safety is ensured.

Transverse reinforcement on the piles:

$$A_{st} = \frac{0.25F_{de}}{f_{yd}} = \frac{0.25x800}{43.5} = 4.60 \text{ cm}^2.$$

## 6. CONCLUSIONS

An iterative strut-and-tie model for designing concrete pile caps is proposed in this work. The methodology used to determine the height of the CCC nodal zone under the column is the main difference between this model and traditional strut-and-tie methods. Several traditional strut-and-tie methods use an arbitrated value $x$ for the height of the CCC nodal zone. In these methods, the value of $x$ is chosen without the use of any methodology, for example, it is simply adopted $x = 0.30d$.

In the proposed model, crushing of the struts below the column is verified on a horizontal plane located into the pile cap. Thus, it is considered that the vertical stress below the column spreads to a depth $x$ where it has been reduced enough not to cause crushing of the struts. The determination of the depth of this horizontal plane (the height of the CCC nodal zone under the column) requires an iterative process.

In a previous article [11], the magnified area under the column was defined considering a load distribution along the height of the pile-cap with the same inclination of the struts. The present paper introduces a modification in the geometry of the magnified area that simplifies the use of the model. With this new formulation for the magnified area, it is possible to elaborate graphs that allow the direct solution of the problem, without the need to perform iterations. Some graphics of this type are shown in Figs. 5 and 6.

The proposed model was used to analyze 186 pile caps tested by other authors, being 37 two-pile caps, 21 three-pile caps and 128 four-pile caps. The partial safety factors method was employed for pile caps design and structural safety was evaluated by means of the reliability index. The small failure probability, estimated from the reliability index, demonstrates the safety of the proposed method. A numerical example of dimensioning was also presented.

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