1. INTRODUCTION

Fiber-reinforced polymer (FRP) bars have emerged as an alternative reinforcement for concrete structures. On the one hand, this kind of reinforcement exhibits such properties as corrosion resistance, electromagnetic neutrality and high cuttability [1]. As a result it can have many applications, especially in structures used in marine environments, in chemical plants, when electromagnetic neutrality is needed, or in temporary structures.

On the other hand, FRP bars have low modulus of elasticity and high tensile strength [2]. Due to their mechanical properties, deflections and cracking of FRP RC flexural members are larger than of traditional RC members. As a result, the design of FRP RC beams is often governed by the serviceability limit states [3]. This paper presents the results of a numerical study in which three GFRP RC beams were tested in four-point bending. The aim of this simulation was to examine the failure mechanism and deflections of sim-
ply supported GFRP RC beams depending on the reinforcement ratio. The dimensions of the specimens and properties of concrete and GFRP bars were assumed on the basis of an experimental study [4]. The results of the numerical simulation were compared with code formulations [2, 5] and with the results of experimental tests [4].

2. NUMERICAL SIMULATION

2.1. Test specimens

The numerical model of beams was created on the basis of the beam which is shown in Fig. 1. The numerical study consisted in investigating the flexural behaviour of three beams with varying GFRP reinforcement (Tab. 1). All beams had a cross-section of 0.14 × 0.19 m², a total length of 2.05 m and a span of 1.80 m. The shear reinforcement consisted of 8 mm round steel stirrups placed at intervals of 70 mm. In the pure bending zone no stirrups were provided. Two 6 mm steel bars were used as top reinforcement to hold the stirrups.

### Table 1.
**Characteristics of specimens**

<table>
<thead>
<tr>
<th>Beam designation</th>
<th>Main bar</th>
<th>Reinforcement ratio $\rho_f$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>…-2#12</td>
<td>2#12</td>
<td>0.99</td>
</tr>
<tr>
<td>…-2#16</td>
<td>2#16</td>
<td>1.77</td>
</tr>
<tr>
<td>…-3#16</td>
<td>3#16</td>
<td>2.66</td>
</tr>
</tbody>
</table>

2.2. Materials properties

Concrete

All beams had a target concrete compressive strength of 30 MPa. The properties of concrete were evaluated from cylindrical specimens. They are presented in Tab. 2.

### Table 2.
**Mechanical properties of concrete**

<table>
<thead>
<tr>
<th>Modulus of elasticity $E_c$ [GPa]</th>
<th>Compressive strength $f_c$ [MPa]</th>
<th>Tensile strength $f_{ct}$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.8</td>
<td>32.1</td>
<td>2.8</td>
</tr>
</tbody>
</table>

GFRP

GFRP ribbed bars were used as a flexural reinforcement. The experimentally determined mechanical properties of reinforcement are shown in Tab 3.

### Table 3.
**Mechanical properties of GFRP reinforcement**

<table>
<thead>
<tr>
<th>Diameter [mm]</th>
<th>Tensile strength $f_{fu}$ [MPa]</th>
<th>Modulus of elasticity $E_f$ [GPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>1321</td>
<td>63.4</td>
</tr>
<tr>
<td>16</td>
<td>1015</td>
<td>64.6</td>
</tr>
</tbody>
</table>

2.3. Model of the beam

The finite element (FE) model of considered beams was implemented in ABAQUS environment [6]. The analysis was performed on 2D model and the following assumptions were adopted:

- Concrete damage plasticity (CDP) model of concrete [7] was assumed,
- Tension stiffening effect was taken into account,
- GFRP reinforcement was assumed as a linear elastic isotropic material,
- Steel reinforcement was assumed as a linear elastic-plastic material with isotropic hardening,
- The reinforcement was modelled as 2-node truss elements embedded in 4-node elements of plane stress (Fig. 2).
The model of beams consisted of two different types of finite element:
• T2D2 – 2-node 2D truss elements,
• CPS4R – 4-node plane stress elements with reduced integration.

The concrete was modelled as concrete damage plasticity material, which is based on the brittle-plastic degradation model [8]. For concrete under uniaxial compression, the stress-strain curve shown in Fig. 3 was adopted. It is composed of the Eurocode 2 [5] parabolic ascending branch and a descending branch extended up to the ultimate strain \( \epsilon_{cu} \) [9].

The tension stiffening effect was taken into account by applying a modified Wang & Hsu [10] formula (Eq. 1) to describe the behaviour of concrete under tension (Fig. 4):

\[
\begin{align*}
\sigma_t = \begin{cases} 
E_c \epsilon_t, & \epsilon_t \leq \epsilon_{cr} \\
 f_{ctm} \left( \frac{\epsilon_{cr}}{\epsilon_t} \right)^n, & \epsilon_t > \epsilon_{cr}
\end{cases}
\end{align*}
\]

where \( E_c \) is the modulus of elasticity of concrete, \( \epsilon_t \) is the tensile strain of concrete, \( \epsilon_{cr} \) is the tensile strain at concrete cracking, \( f_{ctm} \) is the average tensile strength of concrete and \( n \) is the rate of weakening.

According to ACI 440.1R-06 [2], the failure mode is governed by concrete crushing when the reinforcement ratio \( \rho_f \) is greater than the balanced reinforcement ratio \( \rho_{fb} \):

\[
\rho_f = \frac{A_f}{bd}
\]

\[
\rho_{fb} = 0.85 \beta_1 \frac{f_{cu}}{f_{fu}} \frac{E_f \epsilon_{cu}}{E_f + f_{fu}}
\]

where \( A_f \) is the area of GFRP reinforcement, \( b \) is the width of the section and \( d \) is the effective depth. In Eq. (3), \( \beta_1 \) is the ratio of depth of equivalent rectan-
gular stress block to depth of the neutral axis, $f_c$ is the concrete compressive strength, $f_{fu}$ is the design tensile strength of GFRP reinforcement, $E_I$ is the modulus of elasticity of FRP, and $\varepsilon_{cu}$ is the maximum concrete strain (0.003 for ACI 440.1R-06 [2]). The actual and balanced reinforcement ratios are compared in Tab. 4. All the beams had higher reinforcement ratios than $\rho_{fb}$, hence according to code [2], failure by concrete crushing was expected in all of them. This mode of failure was confirmed by the numerical analysis and the results of experiments [4].

<table>
<thead>
<tr>
<th>Actual reinforcement ratio $\rho_f$ [%]</th>
<th>Balanced reinforcement ratio $\rho_{fb}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_f$</td>
<td>$\rho_{fb}$</td>
</tr>
<tr>
<td>2#12</td>
<td>0.99</td>
</tr>
<tr>
<td>2#16</td>
<td>1.77</td>
</tr>
<tr>
<td>3#16</td>
<td>2.66</td>
</tr>
</tbody>
</table>

Experimental (EXP), numerical (FEM) and theoretical (ACI for ACI 440.1R-06 and EC2 for Eurocode 2) ultimate loads are compared in Tab. 5. There is good agreement between the experimental and numerical results, whereas ultimate loads calculated according to the codes [2, 5] are underestimated. Their values are lower than the values of loads obtained in the experimental tests [4] by about 26-31% and 10-15% for ACI and EC2, respectively. These differences can be caused by the value of the maximum concrete compressive strain $\varepsilon_{cu}$ which is assumed in these codes (~0.0030 for ACI and 0.0035 for EC2. The results of experiments [11] show that the actual ultimate concrete strain $\varepsilon_{cu}$ is about 0.0042-0.0047.

On the basis of the results shown in Tab. 5, it can be said that the reinforcement ratio has an influence on the flexural strength of the beams. The increase in the reinforcement ratio results in the increase in the ultimate loads of the beams.

### 3.2. Deflections

Figs. 6-8 show the numerical, theoretical and experimental load-deflection curves for all beams. The results of the numerical analysis correspond well with the results obtained in the experiments.

Comparing theoretical predictions obtained based on ACI (Eq. 4) and EC2 (Eq. 5) with the results of experimental tests, it can be observed that up to the service load (deflection $d<\frac{L}{250}$) there is good agreement between theoretical and actual values of deflections. For higher loads these codes underestimate deflections. These differences can be connected with the fact that these theoretical approaches use a simplified linear stress-strain constitutive relationship for concrete.

$$I_c = \left(\frac{M_{cr}}{M_a}\right)^3 \beta_d I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right]I_{cr} \leq I_g$$ (4)

Eq. 4 shows the expression for an effective moment of inertia $I_c$ of the concrete section according to ACI, where $I_g$ is the gross moment of inertia of concrete section, $I_{cr}$ is the moment of inertia of the cracked section, $M_{cr}$ is the cracking moment, $M_a$ is the maximum moment in the member and $\beta_d$ is the reduction coefficient related to the reduced tension stiffening effect.
Eq. 5 shows the formulation for deflections \( \delta \) according to Eurocode 2, where \( \delta_1 \) is uncracked-state deflection, \( \delta_II \) is fully cracked-state deflection and \( \zeta \) is the coefficient related to the tension stiffening effect.

\[
\delta = \zeta \delta_II + (1 - \zeta) \delta_1
\]  
(5)

As can be observed in Figs. 6-8, the reinforcement ratio has a significant effect on the stiffness of the RC beams. As expected, higher deflections are obtained for lower reinforcement ratios and vice versa.

**4. CONCLUSIONS**

This paper presents the results of numerical, theoretical and experimental study of the flexural behaviour of GFRP RC beams. Based on these results, the following conclusions may be drawn:

- The reinforcement ratio has a significant effect on the flexural behaviour of the GFRP RC beams. The increase in the reinforcement ratio results in the increase in the ultimate loads and in the stiffness of the beams.
- The failure mode is governed by concrete crushing when the reinforcement ratio \( \rho_f \) is greater than the balanced reinforcement ratio \( \rho_{fb} \) (according to ACI 440.1R-06). All beams behave almost linearly up to the moment of failure, which takes place at relatively large deflections.
- At the service load level, the deflections calculated according to ACI 440.1R-06 and Eurocode 2 are in close agreement with the results of the experiments. For higher loads these codes underestimate deflections.
- The ultimate loads calculated according to ACI 440.1R-06 and Eurocode 2 are underestimated. This underestimation can be caused by the value of the ultimate concrete strain \( \varepsilon_{cu} \) which is assumed in these codes. It is lower than the value of \( \varepsilon_{cu} \) obtained in experiments.
- The nonlinear model of concrete, which was adopted in the study, reflects relatively well the behaviour of the actual concrete.

**ACKNOWLEDGMENTS**

The paper was presented at the 8th International Conference AMCM 2014 – Analytical Models and New Concepts in Concrete and Masonry Structures (AMCM’2014), Wroclaw, June 2014.

**REFERENCES**


